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Novel Approaches for the Clearing of the European Day-Ahead Electricity Market

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***Abstract*—In this paper three novel approaches are presented for the clearing of the European day-ahead electricity market, incorporating the majority of the currently tradable products, namely the simple hourly orders, the block orders, the complex orders and the PUN ("Prezzo Unico Nazionale") orders. The first approach employs a Master Problem and a Sub-problem sequentially solved within an iterative process, whereas two single-model approaches are also formulated for the efficient handling of the market orders' clearing conditions and the handling of non-intuitive bilateral exchanges. In the single-model approaches the clearing conditions of the block, complex and PUN orders are explicitly incorporated in the Sub-problem, by utilizing a Mixed Complementarity Problem formulation. The three approaches are compared in terms of solution efficiency (social welfare) and computational requirements. A sensitivity analysis is performed for all three approaches, in order to evaluate the efficiency of the attained solution with respect to the method used for the designation and subsequent elimination of all attained Paradoxically Accepted Orders.**

***Index Terms*— Block and Complex orders, PUN Orders, Mixed Complementarity Problem, Complementarity Conditions, Paradoxically Accepted Orders, Intuitive exchanges**

Nomenclature

The main notation utilized in this paper is presented in this section, whereas additional parameters and variables are defined as needed.

A. Sets and Indices

$t \in \mathcal{T}$ — Set of trading periods within the trading day (typically, the period is one hour)

$z \in \mathcal{Z}$ — Set of bidding zones; $\mathcal{Z}_P \subseteq \mathcal{Z}$ is the set of bidding zones where PUN orders are submitted

$b \in \mathcal{B}$ — Set of supply offers' and demand bids' steps

$s \in \mathcal{S}_z$ — Set of supply offers submitted to bidding zone z, $\mathcal{S}_z \subseteq \mathcal{S}$; $\mathcal{S}_{lg} \subseteq \mathcal{S}$ and $\mathcal{S}_{mic} \subseteq \mathcal{S}$ are the subsets of supply orders that are subject to the Load Gradient and Minimum Income Condition, respectively

$d \in \mathcal{D}_z$ — Set of demand bids submitted to bidding zone z, $\mathcal{D}_z \subseteq \mathcal{D}$; $\mathcal{D}_p \subseteq \mathcal{D}$ is the subset of demand bids that are cleared at the PUN price

$l \in \mathcal{L}$ — Set of interconnections, where $\mathcal{L}_{ac} \cup \mathcal{L}_{dc} = \mathcal{L}$ denote the AC and DC interconnections, respectively; $\mathcal{L}_{rmp} \subseteq \mathcal{L}$ is the subset of DC interconnections that are subject to hourly ramping constraints

$bo \in \mathcal{BO}_z$ — Set of block orders submitted to bidding zone *z*, $\mathcal{BO}_z \subseteq \mathcal{BO}$

$fho \in \mathcal{FHO}_z$ — Set of flexible hourly orders submitted to bidding zone *z*, $\mathcal{FHO}_z \subseteq \mathcal{FHO}$

B. Parameters

P_{xb}^t, Q_{xb}^t — Price-quantity pair of step *b* of entity $x = \{s, d\}$ in trading period *t*, in €/MWh and MWh, respectively

P_{bo}, Q_{bo}^t — Price-quantity pair of block orders *bo*, in €/MWh and MWh, respectively; the quantity may be different in each trading period *t*

P_{fho}, Q_{fho} — Price-quantity pair of flexible hourly order *fho*, in €/MWh and MWh, respectively

$F_{lt}^{min}\left(F_{lt}^{max}\right)$ — Minimum (Maximum) available capacity of interconnection *l* in trading period *t*, in MW

$Flow_l^{H,ini}$ — Power flow in interconnection *l* during the last hour of the previous trading day, in MW

$PTDF_{zz'}^{lt}$ — Power transfer distribution factor of AC interconnection *l* for an energy transfer (exchange) between bidding zone *z* and *z'* in trading period *t*, in p.u.

τ_{lt} — Flow tariff for the utilization of DC interconnection *l* in trading period *t*, in €/MWh

$LG_s^{up}\left(LG_s^{dn}\right)$ — Increase (Decrease) gradient of the supply order *s*, which is subject to the Load Gradient Condition, in MW/min

$LZ_l^{+}\left(LZ_l^{-}\right)$ — Parameter denoting that DC interconnection *l* starts (ends) to bidding zone *z*, if equal to *1*; otherwise, it is equal to 0

$R_{lt}^{up}\left(R_{lt}^{dn}\right)$ — Hourly ramp-up (ramp-down) limit of interconnection *l* power flow in trading period *t*, in MW/h

$loss_l$ — Loss factor of DC interconnection *l*, in %

C. Variables

i. Continuous Variables

$BX_{zz'}^{t}$ — Bilateral exchange (BEX) between bidding zone *z* and bidding zone *z'* in trading period *t*, in MWh

$q_{sb}^t (q_{db}^t)$ — Cleared quantity of step *b* of supply (demand) entity's *s* (*d*) priced offer (bid) in trading period *t*, in MWh

u_{sb}^t — Acceptance ratio of step *b* of the sub-order of supply offer *s* which is subject to a Minimum Income Condition in trading period *t*, in p.u.

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u_s Acceptance ratio of the supply offer *s* which is subject to a Minimum Income Condition, in p.u.

p_{zt} Net position of bidding zone *z* in trading period *t*, in MWh

f_{lt} Power flow in interconnection *l* in trading period *t*, in MW

K_{sb}^{t} Non-negative complement variable of step *b* of the supply offer *s* quantity upper limit constraint for trading period *t*, in €/MWh

K_{db}^{t} Non-negative complement variable of step *b* of the demand bid *d* quantity upper limit constraint for trading period *t*, in €/MWh

K_{bo} Non-negative complement variable of the block order clearing status constraint, in €/MWh

K_{fho} Non-negative complement variable of the flexible hourly order clearing constraint, in €/MWh

K_s Non-negative complement variable of the MIC offer *s* clearing status u_s constraint, in €/MWh

K_{sb}^{t} Non-negative complement variable of the MIC hourly sub-orders' clearing status u_{sb}^{t}, for trading period *t*, in €/MWh

$K_s^{up,t}$ / $K_s^{dn,t}$ Non-negative complement variable of the supply order *s* subject to the Load Gradient condition (increment/decrement), for trading period *t*, in €/MWh

K_{lt} Non-negative complement variable of the power flow constraint in line *l* for trading period *t*, in €/MWh

K_{lt}^{+} / K_{lt}^{-} Non-negative complement variable of the upper/lower power flow limit constraint in line *l* for trading period *t*, in €/MWh

$KR_{lt}^{H,up}$ / $KR_{lt}^{H,dn}$ Non-negative complement variable of the flow ramping up/down constraint of individual line *l* for trading period *t*, in €/MWh

ii. Binary Variables

u_{bo} Clearing status of block order *bo* in the Master Problem – it is non-negative and continuous in the Sub-problem

u_{fho}^{t} Clearing status of flexible hourly order *fho* in trading period *t* in the Master Problem - it is non-negative and continuous in the Sub-problem

I. Introduction

Recent developments in the European Union's energy strategy have led to the integration of day-ahead electricity markets into a fully functioning European market. Traditionally, European Power Exchanges (PXs) incorporated diverse types of supply and demand market orders and market clearing conditions; for this reason, specific pricing rules and clearing conditions have to be incorporated in the new shaped day-ahead electricity market problem, thus increasing its clearing complexity.

Two special market clearing conditions, currently appearing in the pan-European day-ahead electricity market, are (a) the Minimum Income Condition[1] (MIC) in the Iberian market [1] (aiming at the provision of adequate revenues to producers or possibly Demand Response resources), and (b) the postage-stamp charges for the suppliers in the Italian market (all suppliers pay the "National Single Price" – "Prezzo Unico Nazionale" (PUN) – calculated as the weighted average zonal price [2]); their clearing conditions are handled by the European day-ahead market solver through iterative methodologies [3] and further complicate the effective clearing of the day-ahead electricity market. Moreover, the appearance of Paradoxically Accepted block and flexible hourly orders ("fill-or-kill" orders that are accepted even though they entail a negative welfare to the respective participants), further complicate the market clearing process.

Motivated by the above, many researchers have elaborated the pricing rules of block, MIC and PUN orders, and proposed several models for the clearing of the day-ahead market [4]-[10]. Even the creators of the day-ahead electricity market solver have released several updated versions of the solver (Euphemia) in the last four years (2014-2017), in order to handle more efficiently the traded products and the problem constraints[2]. Nevertheless, there is no extensive bibliography on the simultaneous handling of all market products that are tradable in PXs, due to the modeling complexity involved in their market clearing conditions, which requires the definition of both primal and dual variables, taking part in a non-linear and non-convex formulation.

The appearance of Paradoxically Accepted Orders (PAOs) in the market clearing solution has been handled through various methodologies in the existing literature. In reference [4], *Martin et al* utilize an iterative procedure, where the current node of the MIP market clearing algorithm is discarded through two different types of cuts, in case PAOs appear in the current solution of the algorithm. Moreover, *Dourbois and Biskas* [5], *Biskas et al* [6], *Chatzigiannis et al* [7] also provide an iterative methodology, where the identified PAOs are completely removed from the order book, before the re-initialization of the market clearing algorithm.

In contrast to the aforementioned iterative procedures, various researchers have also presented "one-shot" formulations for the handling of PAOs. In [8] *Zak et al* utilized a bilevel approach, where the lower level represented the market clearing process and the upper level was used for the enforcement of positive welfare constraints, whereas *Madani and Van Vyve* [9] prevented the appearance of PAOs in the final market clearing solution through a primal-dual formulation.

Moreover, various researchers have elaborated the

[1] Under the Minimum Income Condition, MIC supply orders must collect based on the current Market Clearing Prices, a revenue, comprising a fixed and a variable term. In case that they fail to collect the request revenue, then they are excluded from the finally attained solution

[2] https://www.entsoe.eu/Documents/Network%20codes%20documents/Implementation/stakeholder_committees/2016_02_03/20160126_PCR_Euphemia_Performance_MESC_FEB_2016_final.pdf

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incorporation of the Minimum Income Condition. *Madani and Van Vyve* [9] incorporate in their prima-dual formulation additional linearized constraints, which enforce the Minimum Income Condition. On the other hand, *Chatzigiannis et al* [7] propose an iterative methodology, where a Minimum Income Condition check is performed after the solution of the market clearing problem and all MIC orders that do not fulfill their clearing condition are removed from the Order Book.

The explicit modeling of the PUN orders' clearing condition has received limited attention from the research community, mainly due to its highly non-linear nature. To the best of the authors' knowledge, only *Vlachos and Biskas* [10] and *Chatzigiannis et al.* [7] have presented market clearing solutions that incorporate the aforementioned type of orders. In [10], the authors utilized a Mixed Complementarity Problem formulation and efficiently solved a market clearing problem with simple hourly supply offers and PUN orders. Exploiting the aforementioned approach [10] *Chatzigiannis et al* [7] presented an iterative methodology for the modeling and clearing of the pan-European day-ahead electricity market. The proposed methodology comprised a welfare-maximizing Master problem, and 24 hourly Mixed Complementarity Problem-based PUN-subproblems, utilized for the determination of the finally cleared PUN orders and market prices.

In this paper, three approaches for the clearing of the pan-European day-ahead electricity market incorporating all type of products and network transmission constraints are presented. In the first approach an iterative algorithm is employed between a Master problem (MP) and a Sub-problem (SP), which are sequentially solved, whereas in the other two approaches the MP and the SP are solved at one-stage; all approaches aim at the efficient handling of all diverse order clearing conditions and transmission constraints. The herein proposed approaches employ different techniques for the handling of:

a) Paradoxically Accepted Orders (PAOs),
b) Non-fulfilled Minimum Income Condition (MIC) orders,
c) Non-intuitive Bilateral Exchanges (BEXs)[3].

The main contributions of the proposed approaches are the following:

a) The incorporation of all different types of market orders (block, flexible hourly, PUN and MIC orders) along with their associated market clearing rules;
b) The handling of the PUN and MIC orders' clearing conditions without the use of an ex-post process, through the explicit incorporation of the complex pricing rules in the clearing problem;
c) The one-shot solution of the market clearing problem (Sub-problem) for all trading periods;
d) The incorporation of FB transmission constraints and the efficient handling of the non-intuitive bilateral exchanges.

The three proposed approaches are evaluated in a 14-zone system case and compared in terms of solution efficiency and computational requirements, whereas a sensitivity analysis is performed for the evaluation of the efficiency of the attained solution.

II. Problem Formulation

In this Section the formulation of the Master Problem (MP) and the Sub-problem (SP) are presented in detail. Even though the MP is executed first during the iterative process (as further explained in Section III.A), this section shall commence with the description of the SP, for achieving better comprehension and clarity of the constituent clearing conditions.

A. Mathematical formulation of Sub-problem

The Sub-problem (SP) is formulated mathematically as an equilibrium problem, where $q_{sb}^t, q_{db}^t, u_{bo}, u_{fho}, u_{sb}^t, u_s$, MCP_z^t constitute the decision variables, comprising the following set of conditions.

1) Definition of prices

The zonal Market Clearing Price (MCP), MCP_z^t, is defined as the complement variable of the zonal net position constraint in each trading period *t*. This variable has practically the same interpretation as the Lagrange Multiplier of the power balance constraint in a classical Linear Programming (LP) formulation.

$$\begin{aligned}&\sum_{d\in D_z}\sum_{b\in B} q_{db}^t - \sum_{s\in S_z}\sum_{b\in B} q_{sb}^t - \sum_{s\in S_{mic}}\sum_{b\in B}\left[u_{sb}^t \cdot Q_{sb}^t\right]\\&-\sum_{bo\in BO_z}\left[u_{bo}\cdot Q_{bo}^t\right]-\sum_{fho\in FHO_z}\left[u_{fho}^t\cdot Q_{fho}^t\right]=p_{zt}\quad \forall z\in\mathcal{Z},\ t\in T\end{aligned}\tag{1}$$

$$\begin{aligned}&\sum_{z'\in\mathcal{Z}}\left[BX_{zz'}^t - BX_{z'z}^t\right]+\sum_{l\in\mathcal{L}_{dc}}\left[LZ_{z,l}^+\cdot\left(fl_{lt}^+ - \left(1-loss_l\right)\cdot fl_{lt}^-\right)\right]\\&-\sum_{l\in\mathcal{L}_{dc}}\left[LZ_{z,l}^-\cdot\left(\left(1-loss_l\right)\cdot fl_{lt}^+ - fl_{lt}^-\right)\right]=p_{zt}\quad\perp MCP_z^t\quad \forall z\in\mathcal{Z}, t\in\mathcal{T}\end{aligned}\tag{2}$$

where fl_{lt}^+, fl_{lt}^- are positive variables, such as $fl_{lt} = fl_{lt}^+ - fl_{lt}^-$ for all DC interconnectors.

Equation (1) provides the net position of bidding zone *z* in trading period *t*. Equation (2) expresses the hourly power balance equation of each bidding zone *z*; the net position of zone *z* is equal to the bilateral exchanges between this zone and all its neighboring zones (connected through AC and DC interconnectors), considering possible losses. It should be noted that in case bidding zones *z* and *z'* are not connected through an AC or DC interconnector, then $BX_{zz'}^t = 0$. It should be noted that in the herein proposed mathematical formulation it is assumed that $Q_{bo}^t, Q_{fho} \geq 0$ for all block and flexible hourly supply offers, respectively, whereas $Q_{bo}^t, Q_{fho} \leq 0$ for all block and flexible hourly demand bids.

2) Simple hourly orders constraints

Condition (3) denotes the upper limits for the clearing of simple hourly supply offers:

$$q_{sb}^t \leq Q_{sb}^t \ \perp\ K_{sb}^t \geq 0 \qquad \forall s\in S, b\in B, t\in\mathcal{T}\tag{3}$$

whereas, the pricing rule complementarity condition associated with the non-negative variable q_{sb}^t is formed as:

$$P_{sb}^t - MCP_z^t + K_{sb}^t \geq 0\ \perp\ q_{sb}^t \geq 0 \quad \forall s\in S_z,\ b\in B,\ t\in\mathcal{T}\tag{4}$$

The clearing conditions and the respective pricing rules for the demand bids are similarly defined as follows:

$$q_{db}^t \leq Q_{db}^t \ \perp\ K_{db}^t \geq 0 \qquad \forall d\in D,\ b\in B, t\in\mathcal{T}\tag{5}$$

[3] A non-Intuitive Bilateral Exchange appears when a bidding zone with a higher clearing price exports energy to a lower-priced neighboring one. Such exchanges appear in the Flow-Based (FB) transmission formulation and increase the overall social welfare, even though they provide "wrong" economic signals to market participants [14].

$$-P_{db}^{t}+MCP_{z}^{t}+K_{db}^{t}\geq 0 \perp q_{db}^{t}\geq 0 \quad \forall d\in D_z,\, b\in B,\, t\in \mathcal{T} \tag{6}$$

3) PUN orders constraints

In condition (6), simple hourly demand bids may be either cleared at the zonal market clearing price of their bidding zone *z*, MCP_z^t, or at the PUN price (only for PUN orders), which is defined as follows:

$$\pi_t=\frac{\sum_{z\in \mathcal{Z}_p}\left[MCP_z^t\cdot \sum_{d\in \mathcal{D}_{pz}}\sum_{b\in \mathcal{B}}\left(q_{db}^t\right)\right]}{\sum_{d\in \mathcal{D}_p}\sum_{b\in \mathcal{B}}q_{db}^t}\quad \forall t\in \mathcal{T} \tag{7}$$

where $\mathcal{D}_{pz}\subseteq \mathcal{D}_p$ is the subset of PUN orders submitted to bidding zone *z*. Thus, the complementarity condition for PUN orders q_{db}^t is formed as:

$$\pi_t+K_{db}^t-P_{db}^t\geq 0 \perp q_{db}^t\geq 0 \quad \forall d\in D_p,\, b\in B,\, t\in \mathcal{T} \tag{8}$$

4) Block Orders and Flexible hourly orders constraints

The block order clearing status condition is defined as:

$$u_{bo}\leq \hat{u}_{bo} \perp K_{bo}\geq 0 \qquad \forall bo\in BO \tag{9}$$

where $\hat{u}_{bo}$ denotes the resulted clearing status of the block order *bo* in the MP (presented in Section II.B), and it is considered in the SP as fixed. The same is valid for all symbols with a circumflex used hereinafter.

The respective price-rule complementarity condition associated with the non-negative variable u_{bo} (for supply) is formed as:

$$P_{bo}-\frac{\sum_t MCP_z^t\cdot Q_{bo}^t}{\sum_t Q_{bo}^t}+K_{bo}\geq 0 \perp u_{bo}\geq 0 \quad \forall bo\in BO_z \tag{10}$$

The second term of the left-hand side of (10) practically represents the weighted average MCP for all trading periods where parameter Q_{bo}^t is non-negative. The respective condition for demand block orders is similar, with a minus sign in the offer price P_{bo} and a plus sign in the weighted average MCP,

Conditions regarding the flexible hourly orders are similar to those of simple hourly orders. Practically the offered quantity is assumed to exist for every trading period of the trading horizon, whereas condition (11) imposes that the total cleared energy (for all trading periods) should not exceed the offered quantity, thus the pricing condition for a supply order *fho* is:

$$P_{fho}-MCP_z^t+K_{fho}\geq 0 \perp u_{fho}^t\geq 0 \qquad \forall fho\in FHO_z,\, t\in \mathcal{T} \tag{11}$$

The condition associated with the clearing of the flexible hourly orders is defined as:

$$\sum_{t\in \mathcal{T}}\left[u_{fho}^t\right]\leq \sum_{t\in \mathcal{T}}\left[\hat{u}_{fho}\right]\perp K_{fho}\geq 0 \quad \forall fho\in FHO_z \tag{12}$$

Again, the respective condition (11) for demand orders *fho* is similar, with a minus sign in the offer price P_{fho} and a plus sign in the MCP, As shown, parameters $\hat{u}_{bo}$ and $\hat{u}_{fho}$ (derived from the current solution/iteration of the MP) actually set the upper limit of the respective variables in the SP.

5) Complex Orders constraints

i. Minimum Income Condition

In the case of the supply offers that are subject to the Minimum Income Condition, the condition associated with the MIC offer clearing status u_s is defined as:

$$u_s\leq 1 \perp K_s\geq 0 \quad \forall s\in S_{mic} \tag{13}$$

The condition associated with the MIC hourly sub-orders' clearing status u_{sb}^t is formulated as follows:

$$u_{sb}^t\leq u_s \perp K_{sb}^t\geq 0 \quad \forall s\in S_{mic},\, b\in B,\, t\in \mathcal{T} \tag{14}$$

Moreover, the price rule complementarity condition associated with the non-negative variable u_s is formulated as follows:

$$VC_s+\frac{FC_s}{\sum_{t\in T}\sum_{b\in B}u_{sb}^t\cdot Q_{sb}^t}-\frac{\sum_{t\in T}\sum_{b\in B}MCP_z^t\cdot u_{sb}^t\cdot Q_{sb}^t}{\sum_{t\in T}\sum_{b\in B}u_{sb}^t\cdot Q_{sb}^t}+K_s\geq 0 \perp u_s\geq 0 \qquad \forall s\in S_{mic},\, z\in Z \tag{15}$$

where VC_s and FC_s denote the variable term and the fixed term of the MIC order *s*, respectively.

The respective complementarity condition associated with the non-negative variable u_{sb}^t of an hourly MIC sub-order is formulated as follows:

$$P_{sb}^t-MCP_z^t+K_{sb}^t\geq 0 \perp u_{sb}^t\geq 0 \qquad \forall s\in S_{mic},\, b\in B,\, t\in \mathcal{T} \tag{16}$$

ii. Load Gradient

The conditions associated with the order *s* that is subject to a Load Gradient Condition (Load Gradient Order) are defined as follows:

$$\sum_{b\in \mathcal{B}}q_{sb}^t-\sum_{b\in \mathcal{B}}q_{sb}^{t-1}\leq 60\cdot LG_{s,t}^{up} \perp K_s^{up,t}\geq 0 \quad \forall s\in S_{lg},\, t\in \mathcal{T}-\{t_1\} \tag{17}$$

$$\sum_{b\in \mathcal{B}}q_{sb}^{t-1}-\sum_{b\in \mathcal{B}}q_{sb}^t\leq 60\cdot LG_{s,t}^{dn} \perp K_s^{dn,t}\geq 0 \quad \forall s\in S_{lg},\, t\in \mathcal{T}-\{t_1\} \tag{18}$$

The respective complementarity conditions associated with the Load Gradient orders' quantity q_{sb}^t are:

$$P_{sb}^t-MCP_z^t+K_{sb}^t-K_s^{up,t+1}+K_s^{dn,t+1}\geq 0 \perp q_{sb}^t\geq 0 \qquad \forall s\in S_{lg},\, b\in \mathcal{B},\, t\in t_1 \tag{19}$$

$$P_{sb}^t-MCP_z^t+K_{sb}^t+K_s^{up,t}-K_s^{dn,t}-K_s^{up,t+1}+K_s^{dn,t+1}\leq 0 \perp q_{sb}^t\geq 0 \quad \forall s\in S_{lg},\, b\in \mathcal{B},\, t\in \mathcal{T}-\{t_1\} \tag{20}$$

Conditions (1)-(20) outline the basic sub-problem formulation for the clearing of the day-ahead electricity market with simple hourly orders, complex orders, PUN orders, block and flexible hourly orders. The above set of conditions is complemented by the AC and DC power flow transmission constraints and all associated ramping constraints.

6) Power Flow Constraints

The power flow constraints are modeled through (21)-(24).

$$f_{lt}=\sum_{z\in \mathcal{Z}}\sum_{z'\in \mathcal{Z}}\left[PTDF_{zz'}^{lt}\cdot BX_{zz'}^t\right] \qquad \forall l\in \mathcal{L}_{AC},\, t\in \mathcal{T} \tag{21}$$

$$f_{lt}=fl_{lt}^{+}-fl_{lt}^{-} \perp K_{lt}\geq 0 \qquad \forall l\in \mathcal{L},\, t\in \mathcal{T} \tag{22}$$

$$f_{lt}\leq F_{lt}^{max} \perp K_{lt}^{+}\geq 0 \qquad \forall l\in \mathcal{L},\, t\in \mathcal{T} \tag{23}$$

$$F_{lt}^{min}\leq f_{lt} \perp K_{lt}^{-}\geq 0 \qquad \forall l\in \mathcal{L},\, t\in \mathcal{T} \tag{24}$$

Equation (21) is used for the modeling of the AC power flows, whereas conditions (23)-(24) express that hourly power flows must be always between its minimum and maximum limit.

7) Power Flow Ramping Constraints

Constraints (25)-(28) express the hourly power flow ramping

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constraints on individual lines [12]:

$$fl_{lt} - Flow_l^{H,ini} \le R_{lt}^{H,up} \quad \perp \quad KR_{lt}^{H,up} \ge 0 \quad \forall\, l \in \mathcal{L}_{rmp}, t \in \{t_1\} \tag{25}$$

$$Flow_l^{H,ini} - fl_{lt} \le R_{lt}^{H,dn} \quad \perp \quad KR_{lt}^{H,dn} \ge 0 \quad \forall\, l \in \mathcal{L}_{rmp}, t \in \{t_1\} \tag{26}$$

$$fl_{lt} - fl_{l,t-1} \le R_{lt}^{up} \quad \perp \quad KR_{lt}^{up} \ge 0 \qquad \forall\, l \in \mathcal{L}_{rmp}, t \in \mathcal{T} - \{t_1\} \tag{27}$$

$$fl_{l,t-1} - fl_{lt} \le R_{lt}^{dn} \quad \perp \quad KR_{lt}^{dn} \ge 0 \qquad \forall\, l \in \mathcal{L}_{rmp}, t \in \mathcal{T} - \{t_1\} \tag{28}$$

The complementarity conditions of variables $BX_{zz'}^{t}$, fl_{lt} and fl_{lt}^{+} and fl_{lt}^{-} are formulated as follows:

$$MCP_z^t - MCP_{z'}^t + \sum_{l \in L} \begin{bmatrix} K_{lt}^{+} - K_{lt}^{-} \\ +KR_{lt}^{up} - KR_{lt}^{dn} \\ +KR_{l,t-1}^{dn} - KR_{l,t-1}^{up} \end{bmatrix} \cdot PTDF_{zz'}^{lt} \ge 0 \perp BX_{zz'}^{t} \ge 0 \qquad \forall\, z, z' \in \mathcal{Z}, t \in \mathcal{T} \tag{29}$$

$$K_{lt} + KR_{lt}^{up} - KR_{lt}^{dn} + KR_{l,t-1}^{dn} - KR_{l,t-1}^{up} = 0 \perp fl_{lt} \quad \forall l \in \mathcal{L}_{DC}, t \in \mathcal{T} \tag{30}$$

$$\sum_{z \in \mathcal{Z}} \left[\left(LZ_l^{z,+} - LZ_l^{z,-} \cdot (1 - loss_l) \right) \cdot MCP_z^t - K_{lt} + \tau_{lt} \right] \ge 0 \perp fl_{lt}^{+} \ge 0 \qquad \forall l \in \mathcal{L}_{DC}, t \in \mathcal{T} \tag{31}$$

$$\sum_{z \in \mathcal{Z}} \left[\left(LZ_l^{z,+} \cdot (1 - loss_l) - LZ_l^{z,-} \right) \cdot MCP_z^t + K_{lt} + \tau_{lt} \right] \ge 0 \perp fl_{lt}^{-} \ge 0 \qquad \forall l \in \mathcal{L}_{DC}, t \in \mathcal{T} \tag{32}$$

The resulting formulation (1)-(32), bearing a square set of variables and complementarity conditions, constitutes a Mixed Complementarity Problem, allowing the incorporation and efficient solution of complex pricing rules.

B. Mathematical formulation of Master Problem

The Master Problem (MP) aims at the maximization of the overall social welfare. The objective function is formulated as follows:

$$\text{Max} \sum_{z \in \mathcal{Z}} \left[\begin{array}{l} \sum\limits_{d \in \mathcal{D}} \sum\limits_{b \in \mathcal{B}} \sum\limits_{t \in \mathcal{T}} \left[P_{db}^t \cdot q_{db}^t \right] - \sum\limits_{s \in \mathcal{S}} \sum\limits_{b \in \mathcal{B}} \sum\limits_{t \in \mathcal{T}} \left[P_{sb}^t \cdot q_{sb}^t \right] \\ - \sum\limits_{bo \in \mathcal{BO}} \sum\limits_{t \in \mathcal{T}} \left[P_{bo} \cdot Q_{bo}^t \cdot u_{bo} \right] - \sum\limits_{fho \in \mathcal{FHO}} \sum\limits_{t \in \mathcal{T}} \left[P_{fho} \cdot Q_{fho} \cdot u_{fho}^t \right] \\ - \sum\limits_{l \in \mathcal{L}_{DC}} \sum\limits_{t \in \mathcal{T}} \left[\tau_{lt} \cdot \left(fl_{lt}^{+} + fl_{lt}^{-} \right) \right] \end{array} \right] \tag{33}$$

The overall social welfare comprises the total load utility minus the total offer cost of all simple hourly[4], block and flexible hourly orders. Moreover, the total welfare depends on the cleared power flow of the merchant DC interconnections, since an additional term is subtracted from the objective function, denoting the congestion rent (welfare loss) due to the related tariffs.

The objective function (33) is subject to power balance constraints (1)-(2), to market constraints (3), (5), (12) (where the upper limit is 1), (13), (17)-(18), to power flow constraints (21)-(24) and to ramping constraints (25)-(28) as presented in Section II.A. The MP constitutes a Mixed Integer Linear Programming optimization problem.

III. Solution Algorithm

In this section the solution algorithms of the three proposed market clearing approaches are analytically described.

A. Iterative algorithm of the 1st approach

The proposed algorithm is graphically presented in Fig. 1 and employs an iterative procedure between the MP and the SP, as follows:

a) Initially, the MP is solved and the clearing status of block and flexible hourly orders are attained. The resulted clearing status of the block and flexible hourly orders are passed to the SP as fixed parameters, denoting the upper limit of the respective (continuous) variables of the SP.
b) The SP is subsequently solved at one-shot and the cleared quantities of all orders, along with the MCPs, are computed for each bidding zone and trading period. The SP is used for the effective handling of PUN and MIC orders, since it explicitly incorporates the clearing conditions of the PUN and MIC orders (equations (7)-(8) and (13)-(16), respectively).
c) The algorithm proceeds with three consecutive checks for the determination of the existence of i) non-intuitive bilateral exchanges, ii) Paradoxically Accepted Orders (PAOs) and iii) MIC and PUN orders with different volumes in the MP and the SP.
d) In case any of the above three checks is not satisfied, then "remedial actions" are taken (as described below) and the algorithm continues with step (a). Otherwise, the algorithm terminates.

In Fig. 1 the symbols with superscript "*mp*" denote the variables of the MP, whereas the superscript "*sp*" denotes variables of the SP. The three above-mentioned checks, along with the respective remedial actions, are analytically presented below.

1) The Intuitiveness Check is performed immediately after the solution of the SP, by calculating the total welfare $W_{zz'}^{t}$ of each bilateral exchange as follows:

$$W_{zz'}^{t} = \left(MCP_{z'}^t - MCP_z^t \right) \cdot \overline{BX}_{zz'}^{t} \qquad \forall\, z \in \mathcal{Z}, z' \in \mathcal{Z}, t \in \mathcal{T} \tag{34}$$

where $\overline{BX}_{zz'}^{t}$ is the optimal value of bilateral exchange $BX_{zz'}^{t}$

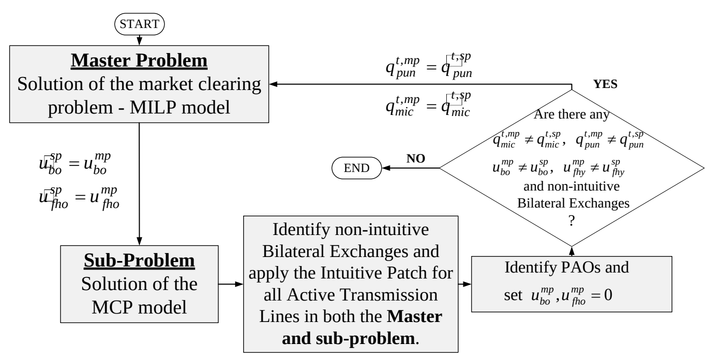

Fig. 1. Schematic illustration of the proposed iterative algorithm

in the current solution of the SP solution. If all exchanges result in intuitive schedules (having a non-negative welfare $W_{zz'}^{t}$), then the algorithm proceeds with the remaining checks. Otherwise, in case at least one bilateral exchange results in negative welfare $W_{zz'}^{t} < 0$ (non-intuitive exchange), then the algorithm designates all congested non-radial AC transmission lines as "*active*" [14] ($l \in \mathcal{L}_{act} \subseteq \mathcal{L}_{AC}$) and replaces equations (23) and (24) with the following constraints:

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[4] Only stepwise hourly orders are considered here for simplicity reasons; the incorporation of linear offers would transform the problem to a Mixed Integer Quadratic Programming (MIQP) model. Such modeling extension is straightforward.

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$$\sum_{z\in\mathcal{Z}}\sum_{z'\in\mathcal{Z}}\left[max(0,PTDF_{zz'}^{lt})\cdot BX_{zz'}^{t}\right]\le F_{lt}^{max}\quad \forall l\in\mathcal{L}_{act},t\in\mathcal{T} \tag{35}$$

$$F_{lt}^{min}\le\sum_{z\in\mathcal{Z}}\sum_{z'\in\mathcal{Z}}\left[min(0,PTDF_{zz'}^{lt})\cdot BX_{zz'}^{t}\right]\quad \forall l\in\mathcal{L}_{act},t\in\mathcal{T} \tag{36}$$

and equation (29) with the following conditions:

$$\begin{aligned}&MCP_z^t-MCP_{z'}^t+\sum_{l\in L_{ord}}\begin{bmatrix}K_{lt}^{+}-K_{lt}^{-}\\ +KR_{lt}^{up}-KR_{lt}^{dn}\\ +KR_{l,t-1}^{dn}-KR_{l,t-1}^{up}\end{bmatrix}\cdot PTDF_l^{zz'}\\ &+\sum_{l\in L_{act}}\begin{bmatrix}K_{lt}^{+}-K_{lt}^{-}\\ +KR_{lt}^{up}-KR_{lt}^{dn}\\ +KR_{l,t+1}^{dn}-KR_{l,t+1}^{up}\end{bmatrix}\cdot max(0,PTDF_{zz'}^{lt})\\ &+\sum_{l\in L_{act}}\begin{bmatrix}K_{lt}^{+}-K_{lt}^{-}\\ +KR_{lt}^{up}-KR_{lt}^{dn}\\ +KR_{l,t+1}^{dn}-KR_{l,t+1}^{up}\end{bmatrix}\cdot min(0,PTDF_{zz'}^{lt})=0\ \perp BX_{zz'}^{t}\ \forall z,z'\in\mathcal{Z},t\in\mathcal{T}\end{aligned} \tag{37}$$

Equations (35) and (36) are applied to all identified "active" transmission lines that are positively and negatively congested, respectively. It should be noted that for all non-congested non-radial AC transmission line constraints (21)-(24) remain the same (the aforementioned lines are designated as "*ordinary*" $l\in\mathcal{L}_{ord}\subseteq\mathcal{L}_{AC}$, according to [14]).

2) After the solution of the MP and the SP the cleared quantities of the MIC and the PUN orders in the respective problems ($q_{sbt}^{mic,mp}/q_{sbt}^{mic,sp}$ and $q_{dbt}^{pun,mp}/q_{dbt}^{pun,sp}$) are attained. If there are observed any differences in the quantities of these orders between the two problems ($q_{sbt}^{mic,mp}\ne q_{sbt}^{mic,sp}$, $q_{dbt}^{pun,mp}\ne q_{dbt}^{pun,sp}$), then the respective variables of the MP in iteration $k+1$ of the algorithm are fixed at the cleared quantities of the SP in iteration k.

3) The handling of the PAOs is performed during the iterative process using equations (9)-(12) of the SP. If the clearing status of the block and flexible hourly orders of the sub-problem is equal to one (or zero), then the respective orders are fully accepted (or rejected). However, if the block order is partially cleared ($0<u_{bo}<1$) or the summation of the flexible hourly orders' status during the trading horizon is between 0 and 1, ($0<\sum_{t\in\mathcal{T}}\left[u_{fho}^{t}\right]<1$) then the respective orders are designated as "Paradoxically Accepted". After the solution of the SP, the status of the identified Paradoxically Accepted orders is set to 0 for the remaining iterations of the algorithm, thus they are effectively removed from the Order Book. A slightly different approach is described in Section IV.C, where, given a certain threshold, block orders may be given more chances to be cleared during the iterative process.

B. Solution algorithm of the single-model approaches

1) Second approach

The second approach involves a single optimization model, in which a Mixed Complementarity Problem similar to the sub-problem of Section II.A is solved. The model comprises the same conditions with the SP, yet with conditions (9) and (12) being substituted by (38)-(39):

$$u_{bo}\le 1\ \perp\ K_{bo}\ge 0\qquad \forall bo\in BO \tag{38}$$

$$\sum_{t\in\mathcal{T}}\left[u_{fho}^{t}\right]\le 1\ \perp\ K_{fho}\ge 0\qquad \forall fho\in FHO \tag{39}$$

An iterative algorithm is also employed in this approach, where ex-post checks are performed only for the identification of (a) Paradoxically Accepted Orders and (b) non-intuitive bilateral exchanges. Paradoxically Accepted Orders and non-intuitive exchanges are handled according to the method described in Section III.A. The model is solved at one-shot (only SP) without containing binary variables, and it comprises conditions (1)-(8), (10)-(11), (13)-(32) and (35)-(39).

2) Third approach

In the third approach, a Mixed Complementarity Problem is formulated as the one presented in the 1st approach, where the non-intuitive exchanges related constraints are incorporated directly in the market clearing problem, according to [11]. In this approach, the non-intuitive exchanges are eliminated endogenously in the model without separating the branches to "active" and "ordinary", as in CWE [14]. The model comprises the same equations with the SP presented in Section II.A, but the bilateral exchange $BX_{zz'}^{t}$ is split in two directional nonnegative components $BX_{zz'}^{t}=BX_{zz'}^{t,+}-BX_{zz'}^{t,-}$ where $BX_{zz'}^{t,+}$ denotes an exchange from the sending zone z to the receiving zone z' and $BX_{zz'}^{t,-}$ the opposite. Moreover, equations (21) and (29) are replaced by the following conditions [11]:

$$f_{lt}=\sum_{z\in\mathcal{Z}}\sum_{z'\in\mathcal{Z}}\left\{PTDF_{zz'}^{lt}\cdot\left[(BX_{zz'}^{t,+})-(BX_{zz'}^{t,-})\right]\right\}\qquad \forall l\in\mathcal{L},t\in\mathcal{T} \tag{40}$$

$$MCP_z^t-MCP_{z'}^t+\begin{Bmatrix}+\sum_{l\in L}PTDF_l^{zz'}|_{>0}\cdot K_{lt}^{+}\\ -\sum_{l\in L}PTDF_l^{zz'}|_{<0}\cdot K_{lt}^{-}\end{Bmatrix}\ge 0\perp\ BX_{zz'}^{t,+}\ge 0\quad \forall z,z'\in\mathcal{Z},t\in\mathcal{T} \tag{41}$$

$$MCP_{z'}^t-MCP_{z}^t+\begin{Bmatrix}-\sum_{l\in L}PTDF_l^{zz'}|_{<0}\cdot K_{lt}^{+}\\ +\sum_{l\in L}PTDF_l^{zz'}|_{>0}\cdot K_{lt}^{-}\end{Bmatrix}\ge 0\ \perp\ BX_{zz'}^{t,-}\ge 0\ \forall z,z'\in\mathcal{Z},t\in\mathcal{T} \tag{42}$$

Equations (40)-(42) are incorporated in the model for the implicit elimination of non-intuitive bilateral exchanges. The model ultimately comprises conditions (1)-(8), (10)-(11), (13)-(32) and (38)-(42). In this case, an iterative algorithm is employed, for the identification and handling of Paradoxically Accepted Orders, following the approach presented in Section III.A).

It should be noted that this model cannot be combined with an initial solution of a MP (MILP model, as described in the 1st approach), since the market clearing conditions (41)-(42) cannot be incorporated in the MP due to the presence of zonal marginal clearing prices and respective complement variables, and non-converging oscillations may possibly occur during the respective iterative algorithm. For this reason, a single-model (one-shot) solution is implemented here, incorporating the MP and SP in a single clearing problem.

C. Differences of the three approaches

The main differences of the three presented approaches are mainly the following:

a) <u>Main difference between the 1st approach and the other approaches</u>: The 1st approach employs a MILP problem (Master Problem) for the determination of the binary variables (clearing status of block orders) and a Mixed Complementarity Problem (Sup-problem) for the

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determination of the orders' cleared quantities and the prices of the solution, solved within an iterative process for the elimination of the paradoxically accepted block and MIC orders and the elimination of non-intuitive exchanges.

The 2nd and 3rd approach constitute single-model approaches, namely they do not bear a Master Problem and a Sub-problem but only a single model (formulated as a Mixed Complementarity Problem), which is solved during an iterative process for the elimination of the paradoxically accepted block and MIC orders and the elimination of non-intuitive exchanges (in the 2nd approach only).

This actually means that binary variables are used in the Master Problem (MILP) of the 1st approach for the simulation of the fill-or-kill properties of the block orders, but the integrality conditions of binary variables are relaxed in the 2nd and 3rd approaches, where block orders are not modeled with binary variables (as in the literature) but with continuous variables. In such modeling, the paradoxically accepted blocks are designated as the partially cleared (accepted) blocks after the solution of the Mixed Complementarity Problem, which are then effectively removed from the Order Book.

b) Main difference between the 2nd approach and the 3rd approach: The 2nd approach eliminates the non-intuitive exchanges by applying the "intuitive patch" (which is applied in CWE). This methodology employs an ex-post check (after the solution of the Mixed Complementarity model) which explicitly modifies the parameters (PTDFs) of the physical electricity grid in order to eliminate the counter-exchanges associated with negative PTDFs in the computation of the (prevailing) flow on the critical/congested branches. Therefore, in the 2nd approach an ex-post check is needed at each iteration, in order to handle the non-intuitive exchanges.

The 3rd approach goes one step beyond (the 2nd approach) and internalizes the non-intuitive exchanges constraints (they are incorporated directly in the market clearing problem), thus there is no need to apply the "intuitive patch" and check for non-intuitive exchanges after the model solution during the iterative process (as in the 2nd approach).

IV. Test Results

A. Case Study

A 14-zone test system is created in order to test the solution efficiency of the proposed approaches. The black lines signify AC interconnections and the orange line a DC interconnection. For demonstration purposes, a set of 9,840 stepwise hourly priced energy offers are artificially created for all bidding zones, out of which 26 are subject to the Minimum Income and Load Gradient Conditions (complex orders, each containing 10 steps for each trading period), along with 3,360 simple hourly demand bids and 1,440 PUN orders. Additionally, a set of 1,050 block orders and 21 flexible hourly orders are also randomly created. The full set of data used in the case study has been made publically available[5] for the efficient reproduction of the herein demonstrated results.

B. Results

Table I presents the results of the iterative algorithm of the 1st approach. As shown, the algorithm converges in five iterations, where all three checks presented in Section III.A are fulfilled. The social welfare decreases in each iteration, as expected. The 3rd column of Table I presents the number of non-intuitive BEXs (between zones connected with an AC interconnection) during the iterative process. As shown, the bulk of non-intuitive BEXs is resolved between the first two iterations, since the enforcement of equations (35) and (36) limits the amount of possible profitable bilateral exchanges, in order to avoid non-intuitive situations (which explains the steep decrease of the total number of bilateral exchanges observed in the second iteration). Finally, columns 4-6 of Table I present the total number of PAOs, MIC and PUN orders with cleared volumes that are different between the MP and SP solutions during the iterative process.

TABLE I
CONVERGENCE OF THE MASTER PROBLEM & SUB-PROBLEM (1ST APPROACH)

Iter	Social welfare (of SP) [€]	Non-intuitive BEXs	PAOs	PUN	MIC
1	148,024,443	85	8	78	13
2	147,487,268	4	6	36	9
3	147,482,548	3	3	5	5
4	147,481,720	0	1	3	1
5	147,481,720	0	0	0	0

TABLE II
CONVERGENCE OF THE 2ND AND 3RD APPROACH

Iter	Social welfare [€]		Non- intuitive BEXs	PAOs	
	2nd	3rd	2nd	2nd	3rd
1	148,026,314	148,031,721	87	14	13
2	147,485,428	148,029,143	6	8	10
3	147,481,898	148,021,867	0	8	7
4	147,481,121	148,021,689	0	3	4
5	147,481,063	148,021,751	0	3	4
6	147,480,935	148,021,552	0	1	2
7	147,480,893	148,021,431	0	0	2
8	-	148,021,326	-	-	0

Table II presents the respective results of the 2nd and 3rd approach. As shown, the social welfare of the 2nd approach decreases sharply in the second iteration, due to the fact that the bulk of non-intuitive BEXs is resolved in the second iteration (4th column) in contrast to the 3rd approach where the non-intuitive exchanges are handled implicitly within the model. There is no column for "inconsistent" PUN and MIC orders in Table II, since the 2nd and 3rd approaches constitute single-model approaches, explicitly incorporating the clearing conditions of PUN and MIC orders.

C. Sensitivity Analysis

In this Section a sensitivity analysis is performed with regard to the designation and subsequent elimination of PAOs in all

[5]https://www.researchgate.net/publication/314205389_Three_models_test_case

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three approaches. Specifically, a threshold is initially defined in the 1st iteration of each run. In case the clearing status of a block order (or a flexible hourly order) is less than the predefined threshold, then the order is designated as PAO and it is rejected; otherwise, the order is given one more chance to be cleared (at the next iteration of the algorithm). The threshold is increased in each subsequent iteration (e.g. by 10%), ultimately reaching 100%, in which case all marginally accepted blocks (in the SP in the 1st approach) are rejected, and the process continues with the next iteration.

A sensitivity analysis is performed for several initial values of the threshold, ranging from 50% to 80% with an increase step of 1% (per scenario), forming 31 scenarios in total. In each scenario, up to four clearing opportunities are given to each PAO. The aim of this analysis is to check the quality of the attained solutions, with respect to the method of designating (and rejecting) PAOs. This sensitivity analysis is performed only for the 1st approach.

In Fig. 2 the clearing results of a specific PAO are presented. Initially, the threshold is set to 65% and is gradually increased by 10% during the first four iterations. As shown, in the first iteration the weighted average MCP (WAP) during the block order period is equal to the block offer price (60.07 €/MWh) and the order is partially cleared. Since the algorithm provides another opportunity for acceptance, the order remains at the Order Book. In the second iteration the order is fully accepted (100%). In the third and fourth iteration, the block order is marginally cleared again (98.88% and 98.5% respectively), as the weighted average MCP is equal to the order price (the threshold is set to 75% and 85%, respectively, in these iterations). Finally, the block order is fully accepted (100%) in the fifth iteration and its clearing status remains the same in all subsequent iterations of the clearing process. Thus, the initially identified PAO becomes finally accepted, due to the provision of additional clearing opportunities in the proposed algorithm.

Fig. 3 presents, in the form of boxplots, the total number of PAOs that are finally accepted (left boxplot) and rejected (middle boxplot). As shown, a small number of PAOs changes their status to “accepted”, due to the provision of additional opportunities, whereas the finally rejected PAOs range between 15 and 25. The presented methodology does not blindly remove PAOs from the Order Book, but gives a certain number of opportunities to each block order that is likely to be cleared at the next iterations to be actually cleared at the final solution. As a result, no good solution (with respect to block orders' clearing) shall be lost, and there is no chance to remove all block orders at the final solution. Moreover, the right boxplot presents the total execution time for all 31 scenarios; as shown, the total execution time ranges between 120 and 210 sec, whereas in most scenarios the final solution is attained in approximately 150 sec.

The aforementioned sensitivity analysis is performed for all three herein-presented approaches; an initial threshold is used according to the first column of Table III, and is gradually increased by 10%, in each subsequent iteration. As shown, the 3rd approach exhibits higher social welfare in all test cases. It should be noted that the social welfare generally decreases as the examined threshold increases, since fewer opportunities are given to PAOs to become accepted. Nevertheless, such differences are small (within each column), indicating that the three approaches provide similar results, regardless of the initial threshold. Moreover, the number of required iterations for convergence is presented in the 2nd column, for each one of the examined approaches.

Finally, Fig. 4 presents the execution time required for the solution of the three presented approaches, with respect to the values of the initial threshold. As shown, the 1st approach converges faster, due to the good initialization of the sub-problem variables, provided by the current solution of the MP,

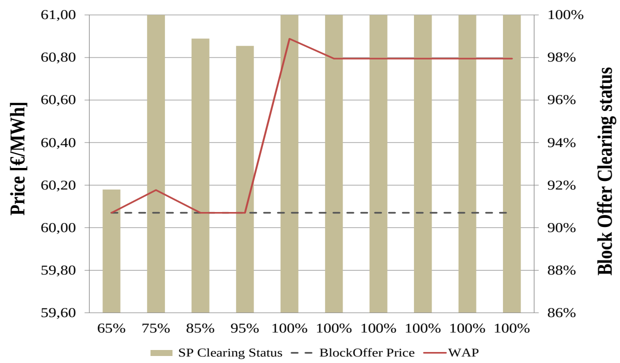

Fig. 2. Block Order clearing with a gradually increasing threshold

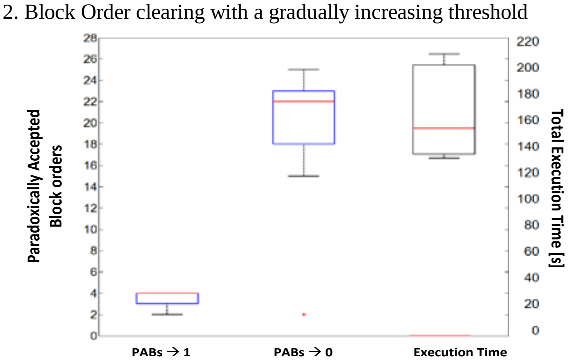

Fig. 3. Range of accepted / rejected PAOs and execution times

TABLE III
SOCIAL WELFARE OF THE THREE PROPOSED APPROACHES

Threshold %	No. of iterations	1st approach [€]	2nd approach [€]	3rd approach [€]
40-100	11/11/10	147,485,815	147,484,532	148,022,172
50-100	11/11/10	147,485,815	147,484,442	148,022,172
60-100	10/10/9	147,485,815	147,481,511	148,022,172
70-100	9/9/9	147,485,289	147,481,511	148,022,184
80-100	9/8/9	147,485,289	147,481,353	148,021,170
100	5/7/8	147,481,720	147,480,891	148,021,170

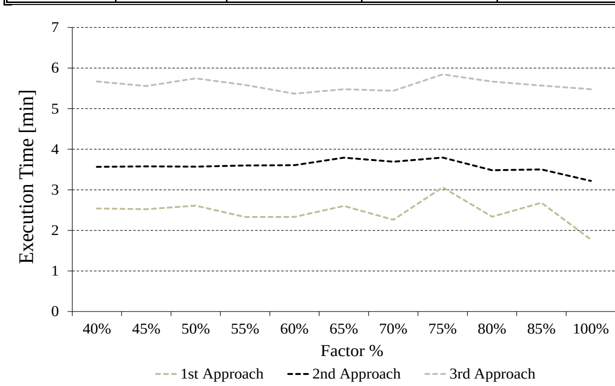

Fig. 4. Execution times of all presented approaches

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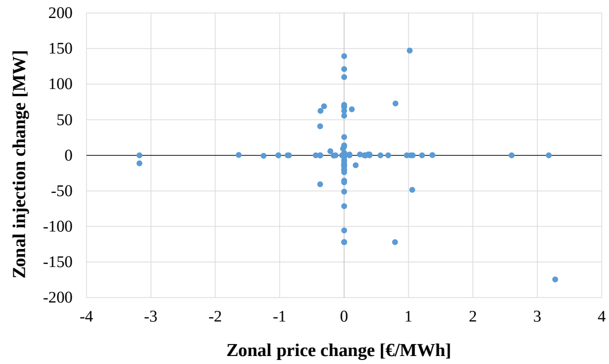

Fig. 5. Zonal price/injection changes between the 1st and 2nd approach

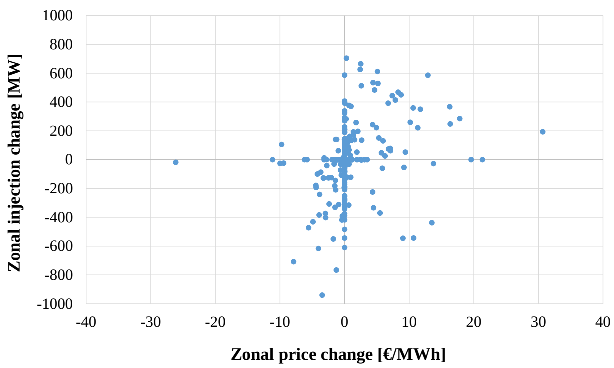

Fig. 6. Zonal price/injection changes between the 2nd and 3rd approach

whereas the 3rd approach is the most time-consuming, due to the inherent difficulties in solving Mixed Complementarity Problems. Nevertheless, in all cases the execution time is less than 10 minutes, which constitutes the widely-accepted, by the European market operators, time-frame for the clearing of the day-ahead electricity market.

D. Comparison of the three approaches' solutions

Fig. 5 and 6 present the perturbation performed in the proposed approaches' solutions, with respect to the zonal injections and prices. Comparing the first two approaches, the 2nd approach results mostly in zonal net position changes (with respect to the 1st approach) under no respective zonal price change or in small zonal price changes under no respective zonal net position change. This is graphically illustrated in Fig. 5, which depicts the correlation of zonal price/injection changes between the first two approaches. The small changes in the cleared quantities / prices are attributed to the fact that in the 1st approach the binary decisions are taken by a separate MILP problem clearing (Master Problem), and then passed to the Sub-problem to attain the continuous variables and the prices, whereas in the 2nd approach a different technique is used for the designation of the clearing statuses of block orders, using purely continuous variables. This procedural difference may lead to the clearing of different blocks, thus to different cleared quantities per bidding zone. Nevertheless, the prices are not significantly affected.

However, the situation is quite different when comparing the second and the third approach (Fig. 6). Although in most cases again there are zonal balance changes under no respective zonal price change or zonal price changes under no respective zonal balance change, still there exist many cases under which both zonal injections and prices differ significantly, expressing the strong perturbation performed in the solution of the 3rd approach, which internalizes the handling of non-intuitive BEXs. As denoted in Section III.C, the 3rd approach attempts to equilibrate an intuitive exchange solution, handling the non-intuitive exchanges endogenously in the model adding either zonal balance cuts or zonal prices cuts that dynamically lead to the maximum usage of critical branches capacity (up to their physical limits) [11]. Thus, this significant procedural difference leads to significantly better social welfare for the 3rd approach, attained through a strong perturbation in the attained results (quantities and prices) with respect to the respective results attained by the 2nd approach. Such procedural difference renders the 3rd approach superior in terms of modeling and attained results with respect to the other two approaches.

E. Computational issues

All cases have been solved in a desktop PC, with an Intel Quad Core i7 CPU processor, running at 3.40 GHz, and 16 GB RAM. The models and the algorithmic process were modeled in the GAMS 24.1.3 modeling environment [15]. The MP has been solved using the CPLEX solver with a relative optimality gap equal to 10^{-7} for all examined cases, whereas the PATH solver has been used for the solution of the SP of the first approach and for the second and third solutions.

The magnitude of the model size of the 1st approach (of the Master Problem and the Sub-problem) is given in Table IV.

TABLE IV
MAGNITUDE OF THE MODEL SIZE OF THE 1ST APPROACH

	Equations	Variables
Master Problem (MP)	17,704	24,482
Sub-Problem (SP)	41,573	41,573

V. DISCUSSION

In this Section the differences between the models proposed in this paper and the models presented in [4] and [6] are highlighted, as follows:

1) First, ref. [4] does not handle PUN orders, which are handled efficiently in all three models presented herein.
2) Second, a MILP problem is presented in ref. [4] employed in an iterative process for the elimination of the paradoxically accepted block and MIC orders. In this paper:
   a) The 1st approach resembles slightly the model presented in ref. [4], since it employs a MILP problem (Master Problem) for the determination of the binary variables (clearing status of block orders) and an MCP model (Sup-problem) for the determination of the orders' cleared quantities and the prices of the solution, solved within an iterative process for the elimination of the paradoxically accepted block and MIC orders and non-intuitive bilateral exchanges.
   b) The 2nd approach is novel as compared to ref. [4] and the other literature, since it employs only an MCP model for the clearing of the day-ahead market. The main point here is that block orders are not modeled with binary variables (as in the literature) but with continuous variables, namely relaxing the integrality constraints (the clearing status of the block can range between [0,1]). In such model, the paradoxically accepted blocks are

designated as the partially cleared blocks, which are effectively removed from the Order Book.

c) The 3rd approach is again novel as compared to ref. [4] and the other literature, since it goes one step beyond (the 2nd approach) and internalizes the non-intuitive exchanges constraints (they are incorporated directly in the market clearing problem), thus there is no need to apply the "intuitive patch" and check for non-intuitive exchanges during the iterative process (as in the 2nd approach). Only an MCP model is also employed in this approach.

3) Third, the clearing conditions of the MIC orders are incorporated within the three models presented herein, and they are not handled (checked for paradoxically accepted MIC orders) using an ex-post process (as in ref. [4]).
4) In ref. [4] the non-intuitiveness of exchanges is not checked at all. In this paper, the non-intuitive bilateral exchanges considered, either explicitly (with ex-post checks in the 1st and 2nd approaches, according to the "intuitive patch" of CWE) or implicitly (internalizing the non-intuitive exchanges constraints in the 3rd approach).
5) In this paper, a sensitivity analysis (with 31 distinct scenarios) is performed for the designation and subsequent elimination of the paradoxically accepted block orders. Using this analysis, the quality of the attained solutions proves better, with respect to the method applied in ref. [4] (of immediately removing the designated paradoxically accepted block orders from the Order Book).

With regard to Ref. [6], the fundamental theoretical differences with regard to the models described herein are the following:

1) In the models described herein, the MIC clearing conditions are incorporated directly within the problem, whereas in [6] an iterative algorithm is followed for the designation and elimination of the paradoxically MIC orders.
2) In [6] a PUN sub-problem is solved in order to enforce PUN market clearing conditions; this PUN sub-problem is coordinated with a Master Problem, taking the binary decisions. Here, in the 2nd and 3rd models there is no coordination between a SP and a MP, since the PUN pricing conditions are incorporated directly within these one-shot models.
3) The model in [6] considers an ATC-based model, ignoring the physical properties of the network and the constituent flows in the critical network elements, whereas in the models described herein the flow-based model is used. The intuitiveness conditions (rendering the 3rd model superior) are active only in the flow-based model.

In order to compare fairly the models, we have compared the model in [6] with the 2nd model described herein applying the ATC-based model, and without giving more clearing opportunities to MIC and block offers. For comparison purposes, three cases are considered:

a) in the first case – *Case BO* – only the block orders are active,
b) in the second case – *Case BO/PUN* – the block orders and the PUN orders are active, whereas
c) in the third case – *Case BO/PUN/MIC* – the block orders, along with the PUN and MIC orders are active.

For the above three cases, the social welfares attained by the 2nd model described herein and by the model in [6] are presented in Table V. As shown, the proposed model outperforms the model [6] in all cases, mainly due to the fact that the 2nd model incorporates all pricing rules and conditions in a one-shot problem, avoiding the division of the problem in two distinct sub-problems, solved sequentially and exchanging optimal solution results to be considered as input in the other sub-problem. The cooperation of these two distinct sub-problems (in [6]) proves deficient with respect to the one-shot problem incorporating all clearing conditions (as implemented in the 2nd model described herein), as expected.

It should be noted that the improvement in the quality of the attained solutions is significantly higher when MIC and especially PUN orders are incorporated. Specifically, in *Case BO/PUN* the difference in the social welfare increases, due to the endogenous handling of MIC orders' clearing conditions in the 2nd model, whereas in *Case BO/PUN/MIC* the difference in the social welfare reaches even higher levels, due to the endogenous handling also of PUN clearing conditions in the one-shot problem.

TABLE V
COMPARISON OF THE SOCIAL WELFARE WITH MODEL IN REF. [6]

Cases	Social Welfare		
	2nd model in this paper	**[6]**	**Comparison [%]**
BO	148,901,948.09	148,898,533.54	0.002293%
BO/PUN	148,880,200.97	148,875,366.26	0.003247%
BO/PUN/MIC	148,127,321.06	147,642,125.94	0.327553%

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VI. CONCLUSIONS

In this paper three novel approaches are presented for the clearing of the European day-ahead electricity market. The modeling and solution methodology of the presented approaches constitutes a significant contribution in the existing literature, since (a) the clearing of block, flexible hourly, MIC and PUN orders, along with the handling of the non-intuitive bilateral exchanges are explicitly incorporated in the market clearing process, and (b) the day-ahead electricity market problem is solved simultaneously for the whole trading horizon. The third approach outperforms the other two approaches, leading to the optimum social welfare due to the straight enforcement of problem-specific complementarity conditions (handling the non-intuitiveness issue) in the associated model, to the detriment of slightly increased computational requirements. The presented analysis and comparison can serve as a *pharos* for future modeling activities of the research community.

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