PV System Control to Provide Active Power Reserves under Partial Shading Conditions

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Abstract—The challenges of modern power systems will inevitably impose increased ancillary service requirements to photovoltaic (PV) plants in the future, including operating reserves. Recent studies investigate methods to maintain active power reserves without energy storage in the standard case of uniform illumination. In this paper, this functionality is extended to partial shading conditions, often encountered in PV systems. A new control scheme is proposed that permits operation at a reduced power level, estimating at the same time the shading conditions and maximum available power. This is achieved by applying a least squares curve fitting algorithm on voltage and current measurements, without relying on any irradiance or temperature sensors. To the best of our knowledge, this is the first power reserves control scheme for PV systems under partial shading presented in the literature. The robustness and effectiveness of the proposed method is validated under rapidly changing shading conditions through simulations and experimental tests on a 2 kW PV system prototype.

Index Terms—Active power control, maximum power point tracking (MPPT), partial shading, power reserves, photovoltaic (PV) system.

I. INTRODUCTION

THE intermittent nature of renewable energy generation imposes technical challenges to the power system operation and control [1], [2]. Network codes are constantly revised, requiring increasingly more services from these systems, including fault-ride-through, reactive power injection, frequency response, operating reserves and output power variability (rate of change) constraints [3]–[9]. To provide some of these services, a power plant should be able to keep active power reserves and track power set-points, permitting effective up and down regulation of its output on command [10]–[12].

In photovoltaic (PV) systems, operating reserves can be provided by either installing energy storage or curtailing generation [5], [12]–[14]. The former option employs batteries [15] ultra capacitors [16] or fuel cells [17] to reutilize the curtailed energy, albeit at an increased installation and maintenance cost. The latter approach involves wasting available energy but is easily implemented using a dump load [18] or, more frequently, by modifying the maximum power point tracking (MPPT) algorithm to operate at a reduced power level [1], [4], [6], [11]–[13], [19]–[25]. This control-based alternative has attracted the interest of researchers lately, as it requires no additional hardware and is therefore considered a cost-effective solution [11]–[13]. Major challenges in implementing these methods include: a) effectively tracking power set-points in dynamic conditions and b) monitoring the maximum power while it changes over time [13].

In order to extend the power reserves requirement to small and medium scale PV plants, where a two-stage string-inverter topology is most often adopted [11], [12], modification of the dc/dc converter control is required [10]. In [11], [12], [19]–[21], MPPT-like control schemes are proposed that involve perturbation of the operating point in discrete voltage steps, thus exhibiting mediocre dynamic response. For a better response, a voltage PI controller is employed in [10], [26], which however proves ineffective in tracking power set-points. To address this issue, a control scheme that involves modification of the P-V curve is presented in [13], which enables direct regulation of the power, rather than the voltage. However, all these studies assume uniform illumination and are not suitable for partial shading conditions.

Furthermore, estimation of the available power is a challenging but important task that allows to keep track of maintained reserves [11]–[13]. In [1], [6], [22], [23], this estimation is performed by means of irradiance/temperature sensors and a mathematical model; although simple, this approach suffers in accuracy due to the inevitable deviations of the actual PV system parameters from datasheet values [11]–[13]. The method presented in [12] requires that a part of the PV plant operates in MPPT mode to monitor the actual maximum power; yet, this approach is suitable only for large PV plants with identical components and conditions. An interesting alternative is proposed in [11], where the system periodically enters MPPT mode, the excess energy being stored in the dc link, thus not affecting the grid-side output; however, increasing the dc link voltage close to its upper limit raises safety concerns. In [10], [13], [21], [24]–[26], a mathematical model is applied on voltage and current measurements to estimate the maximum power: simplified linear/quadratic models are applied on a few samples in [10], [21], [25], while curve fitting algorithms based on the single-diode PV model are employed in [13], [24] to tackle problems caused by noise. Among all aforementioned studies, partial shading conditions are taken
into account only in [1] for microinverter systems; yet, this technique is not applicable to the string-inverter topology, where partial shading gives rise to multiple power peaks on the P-V curve [27]–[29].

In light of these limitations, a power reserves control scheme for two-stage PV systems is proposed in this paper, especially designed for partial shading conditions. A special modification of the multi-peak P-V curve is introduced that allows adoption of a power PI controller to quickly and effectively track power set-points. Furthermore, a new method is proposed to estimate the maximum power while keeping reserves. A least-squares curve fitting algorithm is applied on voltage and current measurements using the PV models derived in [27], [29]–[31] to determine the shading conditions, the MPPs and the entire P-V curve in real-time. This is achieved by slightly perturbing the operating point around the power reference to generate the necessary input data for successful curve fitting; there is no need for periodic MPPT operation, as done in other studies. To the best of our knowledge, this is the first method to provide these estimations from a curtailed power level under partial shading in a purely mathematical way. The proposed control is essentially an improvement of [13] utilizing findings of [31] to permit application under partial shading; the algorithm is developed for PV strings (series-connected modules) illuminated at two different irradiance levels and does not require any additional hardware, such as irradiance or temperature sensors. The reliable and effective operation is validated through simulations in MATLAB/Simulink and experimental tests on a 2 kW PV system operating under very fast-changing shading conditions. The proposed method is an easy way to render a PV system without storage a dispatchable power plant, being able to provide reserves and other ancillary services, such as frequency regulation, to the grid.

The proposed control scheme is described in Section II, followed by the MPPs estimation algorithm in Section III. Simulation and experimental results are discussed in Sections IV and V respectively, while the circuit equations adopted are briefly given in the Appendix.

II. POWER RESERVES CONTROL FOR PARTIAL SHADING

The configuration of a two-stage PV inverter is depicted in Fig. 1. The dc/dc converter, usually a boost converter, regulates the operating point and extracts the power from the PV generator adjusting the duty cycle $D_{\text{conv}}$. The inverter transfers that amount of energy to the grid and maintains a constant dc link voltage by applying a P-Q control algorithm [11], [13], [32].

Conventionally, the control of the dc/dc converter is done through a simple MPPT algorithm. To allow operation at a reduced power level, however, a more sophisticated control scheme is presented in this section, which incorporates five individual subsystems highlighted in colour in Fig. 1. The Reserves module determines the scheduled power $P_{\text{sch}}$ based on a reserves command; this constitutes the power reference $P_{\text{sch},f}$ that the Main control module tracks by regulating the duty cycle $D_{\text{main}}$. The maximum power is continuously estimated by the MPPs estimator block using voltage and current measurements at the PV side. To ensure these measurements are fit for purpose, the Sampling & Ripple control subsystem introduces a perturbation signal $D_{\text{ripple}}$ to create limited oscillation of the operating point. When the shadow first appears, a quick curve scan is performed by the Scan block by varying the duty cycle $D_{\text{scan}}$. The final duty cycle of the converter, $D_{\text{conv}}$, will be either $(D_{\text{main}} + D_{\text{ripple}})$ or $D_{\text{scan}}$, depending on the operating Mode. In the following, these modules are described in detail.

A. Reserves Module

The requested reserves may correspond to a command issued by the grid operator or to the output of an internal control block. Reserves may be either a specific power level (kW) [6], [11], [12], [23] or a fraction of available power $P_{\text{max}}$ [1], [10], [13], [22], [24], [26]. The latter approach is adopted in Fig. 1, as it better reflects the grid codes requirements, e.g. [3]:

$$P_{\text{sch}} = (1 - \text{Reserves})P_{\text{max}}$$

(1)

where Reserves may range from 0 to 1. Alternatively, a direct power set-point $P_{\text{req}}$ can be imposed, upper bounded by $P_{\text{max}}$:

$$P_{\text{sch}} = \min\{P_{\text{req}}, P_{\text{max}}\}$$

(2)

Notably, both approaches need the maximum power $P_{\text{max}}$.

B. Main Control Module

Objective of this subsystem is to regulate the duty cycle $D_{\text{main}}$ in order to track a power reference $P_{\text{sch},f}$, based on
As explained in [13], to permit regulation to this power reference, a single voltage regulator is not sufficient. A need for extra control loops or MPPT-like control arises that may appear: the variation of the system. This study adopts and expands the concept of [11], where a power PI controller is used to regulate a PV power signal \( P_{\text{PV}} \) to the reference \( P_{\text{ref}} \) (Fig. 1). This modification is necessary to establish a monotonic relation between the input (power) and output (duty cycle) of the PI controller, as established in [13] for uniform illumination. It is worth noting that this approach ignores the power conversion losses, which are typically very small; as a result, the actual reserves maintained may be slightly less than the requested value. If extra high reserves accuracy is required, this deviation can be compensated considering the converter efficiency, as explained in [11].

1) Reserves under partial shading: Under partial shading conditions the \( P-V \) curve becomes more complicated and proper modification is quite challenging. An indicative shading scenario is illustrated in Fig. 2, with 2 out of 12 modules of a PV string being shaded (Fig. 2(a)). The respective scenario is illustrated in Fig. 2, with 2 out of 12 modules properly modified (Fig. 1). This modification is necessary to establish a monotonic relation between the input (power) and output (duty cycle) of the PI controller, as established in [13] for uniform illumination. It is worth noting that this approach ignores the power conversion losses, which are typically very small; as a result, the actual reserves maintained may be slightly less than the requested value. If extra high reserves accuracy is required, this deviation can be compensated considering the converter efficiency, as explained in [11].

\[
P_{\text{ref}} = \min\{P_{\text{sh}}, P_{\text{lim}}\}
\]

(3)

2) Modification of the \( P-V \) curve (two irradiance levels): As discussed above, there is a need to derive a monotonic version of the \( P-V \) curve in order to apply a power PI controller. To include both downhill sections in this modification, two individual curves \( P_{\text{mod1}} \) and \( P_{\text{mod2}} \) are introduced (coloured dashed lines in Fig. 3). Essentially, each of these curves correspond to one downhill section extended by descending linear segments using the key operating points:

\[
P_{\text{mod1}} = \begin{cases} P_{\text{mp1}} & , V_{\text{mp1}} < V \leq V_{\text{ip}} \\ P_{\text{ip}} & , V > V_{\text{ip}} \end{cases}
\]

(4)

\[
P_{\text{mod2}} = \begin{cases} P_{\text{mp2}} & , V_{\text{mp2}} < V \leq V_{\text{ip}} \\ P & , V > V_{\text{ip}} \end{cases}
\]

(5)

The final modified power feedback, \( P'_{\text{pv}} \), fed into the PI controller will be either \( P_{\text{mod1}} \) or \( P_{\text{mod2}} \) according to the selection rule:

\[
P'_{\text{pv}} = \begin{cases} P_{\text{mod1}} & , (P_{\text{ref}} > P_{\text{mp2}}) \text{ or } (P_{\text{mp1}} > P_{\text{mp2}}) \\ P_{\text{mod2}} & , \text{otherwise} \end{cases}
\]

(6)

This is graphically illustrated in Fig. 3. If \( P_{\text{ref}} \) lies only in one downhill section \( (P_{\text{ref}} > P_{\text{mp2}} \text{ or } P_{\text{ref}} < P_{\text{ip}}) \), the modified curve that corresponds to that section is assigned to \( P'_{\text{pv}} \) (vertical green or red arrow respectively in Fig. 3); if, however, \( P_{\text{ref}} \) lies in both downhill sections \( (P_{\text{ip}} < P_{\text{ref}} < P_{\text{mp2}}) \), the modified curve that already contains the operating point is selected, comparing the voltage \( V_{\text{pv}} \) to the IP voltage \( V_{\text{ip}} \).
(horizontal green ($P_{mod1}$) or red ($P_{mod2}$) arrow in Fig. 3). This way, a small deadband is established which prevents unnecessary transition of the operating point from one section to another. Combining (4)-(6), a monotonic relation between the input and output of the PI controller of the Main control module is established (Fig. 1).

3) Modification of multi-peak P-V curves: In this section, it is shown how the modification rule can be extended to the general case of multi-peak P-V curves. To facilitate understanding, the indicative case of Fig. 4 is used as an example: there are four MPPs (MPP1-MPP4), three IPs (IP1-IP3) and several possible operating points for a given power reference. Based on the previous discussion, the most preferable operating region lies in the rightmost downhill section that provides the required $P_{ref}$.

However, there is no need to consider all downhill segments in the modification, since a subset of those may suffice for all possible $P_{ref}$ values (e.g. MPP2-IP2 and MPP4-OC sections in Fig. 4). In the general multi-peak case, one has to find the minimum subset of rightmost downhill sections that collectively provide the entire power range; these sections are defined by their bounds: subset of MPPs (SMPPs) and subset of IPs (SIPs). The flowchart of this process is illustrated in Fig. 5(a), where $n$ and $m$ are the total number of MPPs and SMPPs respectively. In Fig. 4, this algorithm results in SMPP1, SMPP2 and SIP1, SIP2.

Thereafter, a modified curve is assigned to each one of the selected sections depending on the SMPP$i$ = ($V_{smp-i}$, $P_{smp-i}$) and SIPi = ($V_{sip-i}$, $P_{sip-i}$):

$$P_{mod-i} = \begin{cases} 
-P_{smp-i}(V-V_{smp-i}) + P_{smp-i}, & V < V_{smp-i} \\
0, & V_{smp-i} \leq V \leq V_{sip-i} \\
P - P_{sip-i}(V-V_{sip-i}) / V_{sip-i} - V_{sip}, & V > V_{sip-i}
\end{cases} \quad (7)$$

Equation (7) is the general expression of (4) and (5) for the multi-peak case, while the resulting $P_{mod1}$ and $P_{mod2}$ are shown in coloured dashed line in Fig. 4 for the study-case P-V curve. Finally, selection of the appropriate modified curve is performed based on the flowchart of Fig. 5(b), which is essentially a general version of (6). It is worth noting that this section is included only for the sake of completeness, as the MPP estimation algorithm presented in the following is designed for the common case of two irradiance levels.

Fig. 4. Modification of multi-peak P-V curves.

Fig. 5. Flowchart of (a) the creation of subsets SMPPs and SIPs, and (b) the selection of the appropriate modified curve.

C. MPPs Estimator Module

The two control subsystems described above require knowledge of the maximum power $P_{max}$ and the four key operating points. To estimate these attributes while operating at a reduced power level, the concept of curve fitting presented in [13] is adopted here and expanded to partial shading conditions. An example is shown in Fig. 6: a mathematical model is fitted to a set of measurements included in a measurement window (green circle markers - dispersion is due to noise), in order to estimate the P-V curve and the key operating points (red colour). The details of this algorithm are given in Section III. The measurement window is created by deliberately introducing an oscillation of the operating point around the reference power (purple dotted line) via the Sampling & Ripple control module.

D. Sampling & Ripple Control Module

This subsystem produces the perturbation signal $D_{ripple}$ superimposed on $D_{main}$ (Fig. 1) in order to generate a small
operating point oscillation. This concept has been used in the past in MPPT algorithms [36], [37] and recently was employed in a power reserves control scheme [13]. According to [13], $D_{\text{ripple}}$ is a triangular perturbation signal of fixed period $t_{\text{conv}}$ and adjustable amplitude (see Fig. 7(a) in the following subsection). A PI controller adjusts the amplitude to regulate the measured ripple $R_c$ to a given reference $R_{\text{ref}}$, as shown in Fig. 1.

In [13], the ripple metric $R_c$ is calculated as the Euclidean distance of the boundary samples within the window, rather than simply the voltage or power ripple; this ensures constant window length on the $P$-$V$ curve regardless of the operating region. This ripple metric works well in principle, but may lead to overestimation under high-noise conditions, when the boundary samples significantly deviate from the actual window bounds. To address this issue, this method is improved here using notch filtering techniques.

As illustrated in Fig. 1, the measured voltage $V_{\text{pv}}$ and power $P_{\text{pv}}$ are filtered applying notch filters at the ripple frequency $f_{\text{conv}}$. The induced ripple is thus filtered out from the raw signals to extract the non-oscillating $V_{\text{pv,f}}$ and $P_{\text{pv,f}}$ used in the Main control module. At the same time, the ripple components $V_{\text{pv,r}}$ and $P_{\text{pv,r}}$ are utilized in the new curve ripple metric introduced in this paper:

$$R_c = \sqrt{\left(\frac{V_{\text{pv,r}}}{V_{\text{nom}}}\right)^2 + \left(\frac{P_{\text{pv,r}}}{P_{\text{nom}}}\right)^2}$$

where $V_{\text{pv,r}}, P_{\text{pv,r}}$ are the rms values of the ripple components and $V_{\text{nom}}, P_{\text{nom}}$ the nominal voltage and power of the PV generator, used for normalization purposes. The new ripple metric provides reliable extraction of the induced perturbation, even in presence of strong multi-spectral noise. This fluctuation introduced in the PV output does not reach the grid side, as it is effectively filtered out by the inverter control [13].

### E. Scan Module

The limited operating point oscillation induced by the Ripple control module always suffices for reliable estimation, except when the shadow first appears and its intensity is completely unknown. At that time, a much wider variation of the operating point takes place to capture the entire $P$-$V$ curve. This procedure, commonly known as curve scan in the literature, is performed only once per shading event, triggered when the shadow extent exceeds a certain limit.

To facilitate understanding, an indicative example is shown in Fig. 7. Normally, the duty cycle of the converter $D_{\text{conv}}$ (blue line in Fig. 7(a)) corresponds to the output of the Main control $D_{\text{main}}$ (red dashed line in Fig. 7(a)) modified by the perturbation signal $D_{\text{ripple}}$ of the Ripple control module. As a result, the PV voltage and current in Fig. 7(b) present limited fluctuation as required.

During the scan, however, the duty cycle $D_{\text{scan}}$ is applied instead (Fig. 7(a)), which varies in a triangular manner from a very low to very high value (0 and 0.8 respectively) within a small time period $t_{\text{scan}}$ to acquire a large part of the characteristic (not the entire curve - see voltage and current variation in Fig. 7(b)); at the last $t_{\text{rst}}$, the duty cycle is restored to its value prior to the scan in order for the system to return to its previous steady state smoothly. It is worth noting that during this process the PV power varies significantly, a power fluctuation that is inevitably reflected to the grid side to some extent. Similar short-term fluctuations are apparent in the majority of MPPT algorithms designed for partial shading conditions.

The timings $t_{\text{scan}}$ and $t_{\text{rst}}$ depend on hardware constraints, mainly the switching device current rating, as well as the input capacitance, cables parasitic inductance etc.; the lower these times, the shortest the duration of the scan, thus less power losses and reduced power fluctuation on the grid side. Since only a large portion of the curve is required (rather than the entire characteristic), there is no need for the operating point to converge to the OC or SC (e.g. voltage and current in Fig. 7(b) do not reach zero); this allows for very small scan timings in the range of tens of milliseconds. For the study-case system of the following sections, $t_{\text{scan}}$ and $t_{\text{rst}}$ can be set as low as 60 ms and 10 ms respectively (see Fig. 7(a)-(b) and Table I in Section IV). It is worth noting that the sampling frequency is high enough to capture several hundreds of measurements during this time ($t_{\text{scan}}/t_s = 60\text{ms}/50\mu\text{s} = 1200$ samples).

### III. REAL-TIME MPP ESTIMATION AT PARTIAL SHADING

To mitigate the power losses due to partial shading, some studies introduce additional equipment or converter topology [38], [39], while others employ sophisticated MPPT algorithms in an attempt to track the global MPP on a multi-peak $P$-$V$ curve [40]–[42]. During the typical MPPT function, the operating point lies fairly close to a power peak most of the time, though maybe a local one, which is very convenient for estimating purposes in model-based MPPTs (e.g. [43]).
contrary, a power reserves control scheme regulates the operating point at a reduced power level, possibly not close to an MPP region; MPPs estimation under these conditions is much more challenging and requires solid theoretical background and a strong optimization algorithm.

A. Mathematical Model

1) Basic concepts: Using as a reference the shading scenario of Fig. 2(a), the four shading parameters are the effective irradiance on the unshaded part $G_{un}$ (pu), the shade ratio $s$ (pu), the common cell temperature $T$ (K) and the shadow extent $n_{sh}$ (pu). These constitute the vector of the unknowns, $x = [G_{un}, s, T, n_{sh}]^T$ (0.8, 0.5, 318.15, 0.167)$^T$ in Fig. 2(a)), that needs to be evaluated; then, the $P-V$ curve and the key operating points of the generator can be derived.

A PV string consists of several modules connected in series, each comprising a few cell strings; the cell string is a group of cells coupled with a bypass diode within a module, usually used as the basic modelling block in partial shading studies [27], [34], [35]. When the PV string operates under uniform illumination, its equivalent circuit is shown in Fig. 8(a) employing the single-diode PV model. When it is illuminated at two different irradiance levels, the equivalent model becomes more complicated, as in Fig. 8(b), comprising the shaded and unshaded parts, each consisting of the single-diode circuit and a bypass diode [31], [44].

2) Circuit equations: In the general case, the voltage of a cell string $V_{cs}(G, T)$ and a bypass diode $V_{bp}(G, T)$ are functions of the operating current, the effective irradiance $G$ and the cell temperature $T$ (detailed expressions are given in the Appendix). The PV string voltage $V_{str}$ is the sum of the illuminated and shaded cell string voltages (Fig. 8(b)):

$$V_{str} = N_{tot} [(1 - n_{sh})V_{un} + n_{sh}V_{sh}]$$  \hspace{1cm} (9)

$$V_{un} = V_{cs}(G_{un}, T)$$  \hspace{1cm} (10)

$$V_{sh} = \begin{cases} V_{cs}(sG_{un}, T) & , V_{sh} \geq 0 \\ V_{bp}(sG_{un}, T) & , V_{sh} < 0 \end{cases}$$  \hspace{1cm} (11)

where $N_{tot}$ is the total number of cell strings within the PV string and $V_{un}, V_{sh}$ is the voltage of a single unshaded/shaded cell string. In (11), $V_{sh}$ effectively becomes equal to the voltage drop on the conducting bypass diode when it is negative [27], [31], [34], [35]. The partial derivatives of $V_{str}$ with respect to $x$ are found using (9)-(11) and $\partial V_{cs}/\partial G, \partial V_{cs}/\partial T$ given in the Appendix.

3) Key operating points equations: Given the shading parameters, MPP1, MPP2, IP and OC are calculated using the following expressions [27], [29]:

$$V_{mp1} = N_{tot} [(1 - n_{sh})V_{mp} - n_{sh} \Delta V_D]$$

$$I_{mp1} = G_{un} I_{mp}$$

$$V_{mp2} = N_{tot} [(1 - n_{sh})(sV_{mp} + (1 - s)V_{oc}) + n_{sh} V_{mp}]$$

$$I_{mp2} = G_{sh} I_{mp} [1 + \lambda (1 - n_{sh})]$$

$$V_{ip} = N_{tot} [(1 - n_{sh})(sV_{mp} + (1 - s)V_{oc}) - n_{sh} \Delta V_D]$$

$$I_{ip} = sG_{un} I_{ac}$$

$$V_{oc} = N_{tot} V_{oc}$$

(12)

where $\Delta V_D$ is the voltage drop on the bypass diode (typical values: 0.7-1.1V) and $\lambda$ is an empirical coefficient (typical values: 0.03-0.06); these values could be fine-tuned on the study-case system using regression on simulation results [27]. Thereafter, the maximum power $P_{max}$ is found as $P_{max} = \max\{V_{mp1}I_{mp1}, V_{mp2}I_{mp2}\}$. The terms $I_{ac}^T, V_{ac}^T, I_{mp}^T$ refer to the cell string short-circuit current, open-circuit voltage, MPP current and MPP voltage at cell temperature $T$ (nominal irradiance) according to [29], [30].

B. Least Squares Curve Fitting Algorithm

Objective of the curve fitting algorithm is to find the parameters vector $x = [G_{un}, s, T, n_{sh}]^T$ that optimally fits the model of Section III.A to the measurement window (Fig. 6). A widely used method for this task is the Levenberg-Marquardt algorithm, according to which the unknown vector is updated at each iteration $k$ using [45]:

$$x^{k+1} = x^k - [H + \Delta diag(H)]^{-1} G$$  \hspace{1cm} (13)

where $H = J^T J$ is the Hermitian matrix, $G = J^T R$ the gradient vector, $J$ the Jacobian matrix, $R$ the residual vector and $\lambda$ a damping factor (here 0.001). $H$ and $G$ are given by:

$$G = \begin{bmatrix} s0001 & s0100 & s0011 \\ s0100 & s1000 & s0100 \\ s0011 & s0100 & s0010 \end{bmatrix}$$

$$H = \begin{bmatrix} s2000 & s1100 & s1010 & s1000 \\ s1000 & s0200 & s0110 & s0100 \\ s0100 & s0110 & s0020 & s0010 \\ s0010 & s0100 & s0010 & s0002 \end{bmatrix}$$

where the auxiliary terms $S_{xyzwr}$ stand for the sums of partial derivatives and residual of the $n$ samples $(V_i, I_i)$ within the window:

$$S_{xyzwr} = \sum_{i=1}^{n} \left( \frac{\partial V_{str}(I_i)}{\partial G_{un}} \right)^x \left( \frac{\partial V_{str}(I_i)}{\partial s} \right)^y \left( \frac{\partial V_{str}(I_i)}{\partial T} \right)^z \left( \frac{\partial V_{str}(I_i)}{\partial n_{sh}} \right)^w (V_{str}(I_i) - V_i)^r$$  \hspace{1cm} (15)

Equation (13) is the step of the unconstrained optimization method. A constrained version of (13) is derived by imposing a limit $\Delta x$ on the maximum change of $x$ over one step:

$$x^{k+1} = x^k - \min \left\{ \Delta x, \max \left\{ -\Delta x, [H + \lambda diag(H)]^{-1} G \right\} \right\}$$  \hspace{1cm} (16)

This arises from the fact that the effective irradiance $G_{un}$ and cell temperature $T$ change relatively slowly [13], while
the shade ratio \( s \) remains almost constant for a particular shading event [46]; on the contrary, the shadow extent \( n_{sh} \) may vary from 0 to 1 within seconds. Applying an appropriate \( \Delta x \) limit, (16) provides more robust estimation under simultaneous changes of the shading conditions.

C. Flowchart

Since the Levenberg-Marquadt is an iterative algorithm, proper segmentation of the computational cost has to be made to permit implementation in a microcontroller (MCU). The flowchart is shown in Fig. 9, consisting of two parts: a) the sample processing that takes place at every sampling period \( t_s = 50\mu s \) (upper part), and b) the shading parameters update performed once per control period \( t_{conv} = 20\text{ms} \) (lower part). Thus, the measurement window length is \( t_{conv}/t_s = 20\text{ms}/50\mu s = 400 \) samples.

The sample processing involves all calculations made for every new sample \((V_i, I_i)\) captured to gradually form the sum terms \( S_{xyzwr} \). When a control period has elapsed (after 400 samples), one iteration (16) or (13) is performed depending on the Mode, in order to update \( x \) and the key operating points. In Normal mode, this iteration is made only when the ripple \( R_c \) is limited (not exceeding \( R_{ref} \) by more than \( \Delta R_c \)), which indicates convergence to a certain power level and appropriateness of the window. The system enters Scan mode when the shadow extent \( n_{sh} \) first becomes more than 10% (“Shadow just appeared?” in flowchart of Fig. 9); in that case, a curve scan is performed and the entire \( P-V \) curve is recorded by executing unconstrained iterations till \( x \) is determined. This scan is made only once upon occurrence of a new shading event, rather than being executed periodically as in several MPPT algorithms; thereafter, the iterations of the algorithm in Normal mode suffice for adaptation to any changes in the shading conditions.

IV. SIMULATION RESULTS

The power circuit and control scheme of Fig. 1 is developed in MATLAB/Simulink for a 2 kW PV system. Parameter values for the system are given in Table I, corresponding also to the experimental setup of Section V. It is worth noting that the switching frequency of the inverter and boost converter are the same and equal to the sampling frequency \( f_s = 20\text{kHz} \), while Space Vector Modulation (SV PWM) has been used as the inverter PWM method (see Fig. 1). In the following, step changes in the reserves command up to 40% are simulated, while the PV string experiences a hypothetical very fast shading event with simultaneous changes on effective irradiance and cell temperature. Furthermore, noise (75 SNR) is superimposed on measurements to emulate real-life conditions.

In Fig. 10(a), the achieved reserves level (blue line) sufficiently matches the requested command (red dashed line) throughout the simulation, except for short-term deviations when the scan is performed (1.5 s) and when the operating point shifts from one downhill section to another (4 and 6.7 s). A more complete picture is given in Fig. 10(b); although the maximum power (purple dashed line) varies strongly due to shading, it is estimated very accurately by the algorithm (green line); hence, the output power \( P_{pv} \) (blue line) effectively tracks its reference (red dashed line), except only for the aforementioned spikes.

The progression of the shading event is illustrated Fig. 10(c). The shadow extent \( n_{sh} \) (red dashed line) reaches 42% in a few seconds, being perfectly monitored throughout the simulation (blue line). The estimated shade ratio \( s \) (green line) becomes meaningful only after the scan at 1.5 s, coinciding thereafter with the actual \( s \) (purple dashed line). Simultaneously with the shading, the effective irradiance \( G_{sun} \) and cell temperature \( T \) change by 100 W/m\(^2\) and 1 °C respectively, as shown in Fig. 10(d); although a small deviation between estimation and actual values does exist, this is not reflected in the estimation of \( P_{max} \) (Fig. 10(b)).

The grid-side power is illustrated in Fig. 10(e): the injected power (blue line) matches the requested value (red dashed line).
line) (except for the discreet spikes), while the apparent ripple is negligible compared to the PV-side power ripple (blue line in Fig. 10(b)). This is better demonstrated in Fig. 10(f): the grid power ripple $R_p$ (yellow line) varies within 0.1-0.3% most of the time, much less than the curve ripple $R_c$ (blue line) that tracks its reference $R_{cref} = 1\%$ (red dashed line). Short-term deviations are apparent when the power reference changes abruptly, halting execution of the estimation algorithm when exceeding the purple dashed line limit.

The main conclusion is that the estimation algorithm works well even under fast simultaneous changes of irradiance and shading conditions; the grid-side power does not present noticeable high-frequency ripple, but short-term spikes are apparent when the scan is performed or the operating point switches downhill section. It is worth noting, however, that such spikes are apparent in all existing MPPTs under varying partial shading conditions, as they need to perform a scan-like variation of the operating point regularly [40]–[42] and suffer from power fluctuation when the shadow extent changes. As a matter of fact, the proposed control’s spikes are normally rarer than these MPPTs, as the scan is performed only once per shading event, rather than periodically or whenever the conditions slightly change. So far, the grid operators have not imposed any restrictions to these power fluctuations, as they do not appear very often. Yet, the PV generation smoothing topic has attracted the researchers interest lately [1], [47]–[49], investigating methods to reduce the PV power variation in the general case.

V. EXPERIMENTAL VALIDATION

The experimental setup used to validate the proposed control in real-life conditions consists of a 2 kW PV string comprising 12 modules connected in series (Fig. 11(a)) connected to a boost converter feeding a resistive load (Fig. 11(b)), rather than an inverter feeding the electric grid, for simplicity. The PV string is subject to outdoor solar irradiance, while the time-varying partial shading conditions are emulated by means of semi-transparent fabric that covers/uncovers the PV...
string using a pulleys-ropes mechanism (Fig. 11(a)) (see also supplemental video). Main characteristics of the converter are given in Table I, with additional information provided in Table II. It is worth noting that the MCU capabilities are typical for converters of this power rating, while the firmware implemented runs smoothly, leaving the MCU idle for more than 50% of the time.

In order to test the proposed control scheme, first the parameters of the study-case system need to be determined; this is achieved by performing some special scans (Test#1), as explained in Section V.A. Then, on the following day (different operating conditions) the main experiment takes place (Test#2) to validate the power reserves control (Section V.B).

### A. Test #1: Extraction of the Reference Parameters

As explained in the Appendix, in order to apply the curve fitting equations, seven coefficients must be known: the five parameters \( I_{ph0}, I_{0}, a_0, R_s, R_{sh} \) and the two coefficients of the two bypass diodes \( a_{bp}, I_{shp} \). The best way to extract these parameters is from measurements on the study-case PV system [13]. To this end, two special curve scans, denoted as reference scans, are performed to capture the \( I-V \) curve at uniform illumination and partial shading conditions (Fig. 12). These scans take place much more slowly (10 s in this test) for two reasons: (a) here the entire characteristic is required, which entails larger duration to allow for convergence of the operating point to the OC and SC regions, and (b) there is no time restriction for the reference scans, as they are carried out very rarely (i.e. once per month or year) only to account for long-term changes on the PV system [13]. The dotted curves in Fig. 12 consists of 500 samples each.

First the five parameters are extracted from the unshaded characteristic (blue dots) using the Phangs method [13], [50]; then these are used to find the two bypass diodes’ coefficients applying least squares curve fitting on the shaded curve (green dots). As shown in Fig. 12, the \( I-V \) characteristics reconstructed afterwards using the parameters identified (dashed lines) almost coincide with the scanned curves (dots). The seven coefficients extracted this way are used later on as the reference parameters in the main experiment (Test#2), even though the reference conditions \( G_0 \) and \( T_0 \) differ from the STC. In other words, the MPP estimation algorithm does not really need the STC parameters; any consistent set of the seven parameters at reasonable irradiance and temperature suffice. As shown in the following section, limited inaccuracies at the identification of the reference parameters may lead to estimation errors during the normal control function, but not convergence issues or control instability.

### B. Test #2: Evaluation of the Power Reserves Control

During Test#2, 4 out of 12 PV modules are shaded and un-shaded within 30 s while the system maintains 20% reserves. This case study corresponds to a fast-moving cloud, emulated using the semi-transparent fabric (see the supplemental video for this experiment). The shading is emulated twice within a few minutes under high sun elevation and clear sky conditions: first, the power reserves control is applied and the performance is recorded; then, it is repeated while the converter simply scans the \( P-V \) curve at the various stages of the shading, instead of the normal control function. The measurements in the latter iteration (referred to as actual in the following) are used as a benchmark for the power reserves control evaluation, as the irradiance and temperature have not practically changed in such a short time period during these conditions. Since the shade ratio (fabric transparency) is known, the shadow detection part of the MPPs estimation algorithm is skipped for simplicity in this experiment.

In Fig. 13(a), operating reserves (blue line) match satisfactorily the requested level (red dashed line), with brief excursions...
each time a module is shaded or unshaded due to the abrupt change of the $P$-$V$ characteristic (one spike per module). However, these fluctuations do not reach the output of the PV system, as further explained in the following.

In Fig. 13(b), the estimated $P_{\text{max}}$ (green line) approximates very well the actual value (purple dashed line), even though it varies significantly within a few seconds. As a result, the power at the PV side $P_{pv}$ (blue line) tracks effectively the reference $P_{sch}$ (red dashed line), although it still presents the aforementioned spikes. The operating voltage $V_{pv}$ (PV side) is shown in blue colour in Fig. 13(c) along with the estimated (continuous lines) and actual (dashed lines) MPPs voltages. It is evident that $V_{pv}$ is always between $V_{mp1}$ and $V_{mp2}$, indicating operation at the leftmost downhill section of the $P$-$V$ curve according to the modification rules of Section II.B. Both $V_{mp1}$ and $V_{mp2}$ estimations are quite good, while the $V_{mp2}$ value (yellow line) is meaningless when the system is unshaded (first few and last seconds). In Fig. 13(d), the curve
ripple $R_c$ (blue line) effectively tracks its reference $R_{cs,ref}$ (red dashed line), presenting only limited fluctuation below the yellow dashed line, thus not halting the algorithm execution at any point.

The progression of the shadow is illustrated in Fig. 13(e): the estimation algorithm keeps track of the shadow extent $n_{sh}$ as it varies from 0% to 33% (4 shaded modules out of 12) within a few seconds. The estimated irradiance $G_{est}$ normalized over the reference $G_0$ is shown in Fig. 13(f) in blue colour, ranging around 0.9-0.95; similarly, the estimated temperature $T$ is found approximately 1-1.5 °C higher than $T_0$ (red line). These deviations indicate different operating conditions (Test#2) from the reference ones (Test#1), while the fluctuations on the estimations are reasonable and seem to partially counterbalance each other; as a matter of fact, this partially demonstrates the reliability of the method to a wider operating range close to the reference conditions.

In Fig. 13(g), the estimated (continuous lines) and actual (dashed lines) power of the two MPPs is illustrated; it is apparent that MPP1 is the global maximum the whole time, being monitored very accurately, whereas the weaker MPP2 is approximated with larger, albeit acceptable, error. The overall accuracy is shown in Fig. 13(h): the estimated $P-V$ curves (red lines) at various stages of the shading event ($n_{sh} = 0 – 33\%$) are in good agreement with the scans performed afterwards (blue dots) over the entire operating range, confirming the wide-range capabilities of the proposed technique.

To evaluate the impact of the observed spikes, the oscillograms of the input (PV side) and output (dc link) current and power of the converter are illustrated in Fig. 14(a)-(b). The changes of the $P-V$ curve along the course of the shading event are reflected mainly in the current, which experiences short-term decreases when a module is shaded and increases when it becomes unshaded (blue line in Fig. 14(a)). Yet, these spikes are significantly smoothed out at the dc link level (red line in Fig. 14(a)), as they are effectively filtered out by the inductance and capacitances of the converter. The overall effect on the power is demonstrated in Fig. 14(b): the output power (dc link - red line) proves much smoother compared to the input power (PV side - blue line).

Overall, the experiments performed validate the effectiveness of the proposed controller applied to an actual PV system under the most demanding partial shading conditions.

**VI. CONCLUSION**

In this paper, a new control strategy for two-stage PV systems is proposed, which succeeds in maintaining operating reserves at partial shading conditions, without energy storage. The maximum power available, as well as the entire $P-V$ characteristic, are estimated in real-time applying curve fitting on voltage and current measurements, dispensing thus with the need for irradiance/temperature sensors or other additional hardware. Simulation results and experimental tests on a 2 kW prototype PV system validate the accuracy and robustness of the proposed control at very fast shading events.

This is a simple way for a PV system to maintain power reserves and other ancillary services, without installing energy storage or other additional equipment. The applicability of the method to a typical PV system is verified, as it has been fully implemented in a standard MCU of moderate capabilities. Future work includes extension of the algorithm to more complicated shading scenarios of three or more irradiance levels, addressing the occasional abrupt spikes on the power output, and utilization of the reserves method in a general ancillary services control scheme with generation smoothing capabilities.

**APPENDIX**

The single-diode model shown in Fig. 8(a) involves the so-called five parameters $\{I_{ph,0}, I_{s,0}, \alpha, R_s, R_{sh,0}\}$, hereafter referring to a *single cell string*. To determine these parameters, one has first to extract the reference values $\{I_{ph,0}, I_{s,0}, \alpha, R_s, R_{sh,0}\}$ at STC and then translate them to the study-case conditions $G$ and $T$, as detailed in [30]. Using the Lambert $W$ function, the voltage of a cell string $V_{cs}$ can be written as an explicit function of the current, the reference parameters and the conditions $G$ and $T$ [27], [30], [44]:

$$V_{cs}(G, T) = R_{sh,0} \{ I_{ph,0} [1 + \alpha I_{sc}(T - T_0)] - I / G \} - R_s I - \alpha I / T_0 w_{cs}$$

$$w_{cs} = W \left\{ \frac{I_{s,0} R_{sh,0} (T/T_0)^2}{\alpha_0 G} \right\} \times$$

$$e^{1.71 \left( 1 - \frac{T}{T_0} \right)} + \frac{R_{sh,0} \{ I_{ph,0} [1 + \alpha I_{sc}(T - T_0)] - I / G \}}{\alpha_0 T / \alpha_0}$$

where $w_{cs}$ is an auxiliary term and the Lambert $W$ function $W \{ \} \}$ is calculated using the series expansions provided in
Then the partial derivatives of \( V_{cs} \) with respect to \( G \) and \( T \) are given by:

\[
\frac{\partial V_{cs}(G, T)}{\partial G} = \frac{R_{sh0} I/F + a_{0} w_{cs} T_{0} (1 + w_{cs}) G}{(1 + w_{cs}) G} \tag{19}
\]

\[
\frac{\partial V_{cs}(G, T)}{\partial T} = -a_{0} w_{cs} + R_{sh0} I_{ph0} T_{0} \frac{cs}{1 + w_{cs}} \times \frac{1 + w_{cs}}{G} - \frac{R_{sh0} I_{ph0} [1 + a_{1sc} (T - T_{0})]}{G} \tag{20}
\]

As for the bypass diode, the derived equation involves the two coefficients \( a_{sbp} \) and \( I_{sbp} \) \([27]\):

\[
V_{bp}(G, T) = a_{sbp} W \left\{ I_{sbp} R_{sh0} e^{-\frac{R_{sh0} I_{ph0} [1 - R_{sh0} I_{sbp} T_{0} + (1 + w_{cs}) (T - T_{0})]}{a_{sbp} G}} - R_{sh0} G \right\} \tag{21}
\]

where the partial derivatives with respect to the operating conditions are assumed to be zero.

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